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## Sommario/riassunto

This book lays the foundation for a theory of coarse groups: namely, sets with operations that satisfy the group axioms “up to uniformly bounded error”. These structures are the group objects in the category of coarse spaces, and arise naturally as approximate subgroups, or as coarse kernels. The first aim is to provide a standard entry-level introduction to coarse groups. Extra care has been taken to give a detailed, self-contained and accessible account of the theory. The second aim is to quickly bring the reader to the forefront of research. This is easily accomplished, as the subject is still young, and even basic questions remain unanswered. Reflecting its dual purpose, the book is divided into two parts. The first part covers the fundamentals of coarse groups and their actions. Here the theory of coarse homomorphisms, quotients and subgroups is developed, with proofs of coarse versions of the isomorphism theorems, and it is shown how coarse actions are related to fundamental aspects of geometric group theory. The second part, which is less self-contained, is an invitation to further research,

where each thread leads to open questions of varying depth and difficulty. Among other topics, it explores coarse group structures on set-groups, groups of coarse automorphisms and spaces of controlled maps. The main focus is on connections between the theory of coarse groups and classical subjects, including: number theory; the study of bi-invariant metrics on groups; quasimorphisms and stable commutator length; groups of outer automorphisms; and topological groups and their actions. The book will primarily be of interest to researchers and graduate students in geometric group theory, topology, category theory and functional analysis, but some parts will also be accessible to advanced undergraduates.

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