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Nota di contenuto	Intro -- Acknowledgments -- Contents -- Notation -- 1 Overview -- 1.1 Setup and Aim -- 1.2 The Obstructions to the Local Limit Theorems -- 1.3 How to Show that the Obstructions Do Not Occur -- 1.4 What Happens When the Obstructions Do Occur -- 1.4.1 Lattice Case -- 1.4.2 Center-Tight Case -- 1.4.3 Reducible Case -- 1.5 Some Final Words on the Setup of this Work -- 1.6 Prerequisites -- 1.7 Notes and References -- 2 Markov Arrays, Additive Functionals, and Uniform Ellipticity -- 2.1 The Basic Setup -- 2.1.1 Inhomogeneous Markov Chains -- 2.1.2 Inhomogeneous Markov Arrays -- 2.1.3 Additive Functionals -- 2.2 Uniform Ellipticity -- 2.2.1 The Definition -- 2.2.2 Contraction Estimates and Exponential Mixing -- 2.2.3 Bridge Probabilities -- 2.3 Structure Constants -- 2.3.1 Hexagons -- 2.3.2 Balance and Structure Constants -- 2.3.3 The Ladder Process -- 2.4 - Step Ellipticity Conditions -- *2.5 Uniform Ellipticity and Strong Mixing

Conditions -- 2.6 Reduction to Point Mass Initial Distributions -- 2.7  
 Notes and References -- 3 Variance Growth, Center-Tightness, and the  
 Central Limit Theorem -- 3.1 Main Results -- 3.1.1 Center-Tightness  
 and Variance Growth -- 3.1.2 The Central Limit Theorem and the Two-  
 Series Theorem -- 3.2 Proofs -- 3.2.1 The Gradient Lemma -- 3.2.2  
 The Estimate of  $\text{Var}(S_N)$  -- 3.2.3 McLeish's Martingale Central Limit  
 Theorem -- 3.2.4 Proof of the Central Limit Theorem -- 3.2.5  
 Convergence of Moments -- 3.2.6 Characterization of Center-Tight  
 Additive Functionals -- 3.2.7 Proof of the Two-Series Theorem -- \*3.3  
 The Almost Sure Invariance Principle -- 3.4 Notes and References -- 4  
 The Essential Range and Irreducibility -- 4.1 Definitions and Motivation  
 -- 4.2 Main Results -- 4.2.1 Markov Chains -- 4.2.2 Markov Arrays --  
 4.2.3 Hereditary Arrays -- 4.3 Proofs -- 4.3.1 Reduction Lemma --  
 4.3.2 Joint Reduction.  
 4.3.3 The Possible Values of the Co-Range -- 4.3.4 Calculation of the  
 Essential Range -- 4.3.5 Existence of Irreducible Reductions -- 4.3.6  
 Characterization of Hereditary Additive Functionals -- 4.4 Notes and  
 References -- 5 The Local Limit Theorem in the Irreducible Case -- 5.1  
 Main Results -- 5.1.1 Local Limit Theorems for Markov Chains -- 5.1.2  
 Local Limit Theorems for Markov Arrays -- 5.1.3 Mixing Local Limit  
 Theorems -- 5.2 Proofs -- 5.2.1 Strategy of Proof -- 5.2.2  
 Characteristic Function Estimates -- 5.2.3 The LLT via Weak  
 Convergence of Measures -- 5.2.4 The LLT in the Irreducible Non-  
 Lattice Case -- 5.2.5 The LLT in the Irreducible Lattice Case -- 5.2.6  
 Mixing LLT -- 5.3 Notes and References -- 6 The Local Limit Theorem  
 in the Reducible Case -- 6.1 Main Results -- 6.1.1 Heuristics and Warm  
 Up Examples -- 6.1.2 The LLT in the Reducible Case -- 6.1.3  
 Irreducibility as a Necessary Condition for the Mixing LLT -- 6.1.4  
 Universal Bounds for  $\text{Prob}[S_N - zN(a,b)]$  -- 6.2 Proofs -- 6.2.1  
 Characteristic Functions in the Reducible Case -- 6.2.2 Proof of the LLT  
 in the Reducible Case -- 6.2.3 Necessity of the Irreducibility  
 Assumption -- 6.2.4 Universal Bounds for Markov Chains -- 6.2.5  
 Universal Bounds for Markov Arrays -- 6.3 Notes and References -- 7  
 Local Limit Theorems for Moderate Deviations and Large Deviations --  
 7.1 Moderate Deviations and Large Deviations -- 7.2 Local Limit  
 Theorems for Large Deviations -- 7.2.1 The Log Moment Generating  
 Functions -- 7.2.2 The Rate Functions -- 7.2.3 The LLT for Moderate  
 Deviations -- 7.2.4 The LLT for Large Deviations -- 7.3 Proofs -- 7.3.1  
 Strategy of Proof -- 7.3.2 A Parameterized Family of Changes of  
 Measure -- 7.3.3 Choosing the Parameters -- 7.3.4 The Asymptotic  
 Behavior of  $V^{0365VN}(S_N)$  -- 7.3.5 Asymptotics of the Log Moment  
 Generating Functions -- 7.3.6 Asymptotics of the Rate Functions.  
 7.3.7 Proof of the Local Limit Theorem for Large Deviations -- 7.3.8  
 Rough Bounds in the Reducible Case -- 7.4 Large Deviations  
 Thresholds -- 7.4.1 The Large Deviations Threshold Theorem -- 7.4.2  
 Admissible Sequences -- 7.4.3 Proof of the Large Deviations Threshold  
 Theorem -- 7.4.4 Examples -- 7.5 Notes and References -- 8  
 Important Examples and Special Cases -- 8.1 Introduction -- 8.2 Sums  
 of Independent Random Variables -- 8.3 Homogenous Markov Chains  
 -- \*8.4 One-Step Homogeneous Additive Functionals in  $L^2$  -- 8.5  
 Asymptotically Homogeneous Markov Chains -- 8.6 Equicontinuous  
 Additive Functionals -- 8.7 Notes and References -- 9 Local Limit  
 Theorems for Markov Chains in Random Environments -- 9.1 Markov  
 Chains in Random Environments -- 9.1.1 Formal Definitions -- 9.1.2  
 Examples -- 9.1.3 Conditions and Assumptions -- 9.2 Main Results --  
 9.3 Proofs -- 9.3.1 Existence of Stationary Measures -- 9.3.2 The  
 Essential Range is Almost Surely Constant -- 9.3.3 Variance Growth --  
 9.3.4 Irreducibility and the LLT -- 9.3.5 LLT for Large Deviations -- 9.4

Notes and References -- A The Gärtner-Ellis Theorem in One Dimension -- A.1 The Statement -- A.2 Background from Convex Analysis -- A.3 Proof of the Gärtner-Ellis Theorem -- A.4 Notes and References -- B Hilbert's Projective Metric and Birkhoff's Theorem -- B.1 Hilbert's Projective Metric -- B.2 Contraction Properties -- B.3 Notes and References -- C Perturbations of Operators with Spectral Gap -- C.1 The Perturbation Theorem -- C.2 Some Facts from Analysis -- C.3 Proof of the Perturbation Theorem -- C.4 Notes and References -- References -- Index.

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Sommario/riassunto

This book extends the local central limit theorem to inhomogeneous Markov chains whose state spaces and transition probabilities are allowed to change in time. Such chains are used to model Markovian systems depending on external time-dependent parameters. It develops a new general theory of local limit theorems for additive functionals of Markov chains, in the regimes of local, moderate, and large deviations, and provides nearly optimal conditions for the classical expansions, as well as asymptotic corrections when these conditions fail. Applications include local limit theorems for independent but not identically distributed random variables, Markov chains in random environments, and time-dependent perturbations of homogeneous Markov chains. The inclusion of numerous examples, a comprehensive review of the literature, and an account of the historical background of the subject make this self-contained book accessible to graduate students. It will also be useful for researchers in probability and ergodic theory who are interested in asymptotic behaviors, random walks in random environments, random dynamical systems and non-stationary systems.

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