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Sommario/riassunto

The purpose of this book is to present the current state of the art of the Virtual Element Method (VEM) by collecting contributions from many of the most active researchers in this field and covering a broad range of topics: from the mathematical foundation to real life computational applications. The presents recent advances in theoretical and computational aspects of VEMs, discussing the generality of the meshes suitable to the VEM, the implementation of the VEM for linear and nonlinear PDEs, and the construction of discrete hessian complexes.
