

1. Record Nr.	UNINA9911150865203321
Autore	Sagan Hans
Titolo	Space-Filling Curves / / by Hans Sagan
Pubbl/distr/stampa	New York, NY : , : Springer New York : , : Imprint : Springer, , 1994
ISBN	1-4612-0871-8
Edizione	[1st ed. 1994.]
Descrizione fisica	1 online resource (XV, 194 p.)
Collana	Universitext, , 2191-6675
Classificazione	54F50 28A75 54-03 01A55 01A60
Disciplina	516.3/62
Soggetti	Geometry
Lingua di pubblicazione	Inglese
Formato	Materiale a stampa
Livello bibliografico	Monografia
Note generali	Bibliographic Level Mode of Issuance: Monograph
Nota di bibliografia	Includes bibliographical references and index.
Nota di contenuto	1. Introduction -- 1.1. A Brief History of Space-Filling Curves -- 1.2. Notation -- 1.3. Definitions and Netto's Theorem -- 1.4. Problems -- 2. Hilbert's Space-Filling Curve -- 2.1. Generation of Hilbert's Space-Filling Curve -- 2.2. Nowhere Differentiability of the Hilbert Curve -- 2.3. A Complex Representation of the Hilbert Curve -- 2.4. Arithmetization of the Hilbert Curve -- 2.5. An Analytic Proof of the Nowhere Differentiability of the Hilbert Curve -- 2.6. Approximating Polygons for the Hilbert Curve -- 2.7. Moore's Version of the Hilbert Curve -- 2.8. A Three-Dimensional Hilbert Curve -- 2.9. Problems -- 3. Peano's Space-Filling Curve -- 3.1. Definition of Peano's Space-Filling Curve -- 3.2. Nowhere Differentiability of the Peano Curve -- 3.3. Geometric Generation of the Peano Curve -- 3.4. Proof that the Peano Curve and the Geometric Peano Curve are the Same -- 3.5. Cesaro's Representation of the Peano Curve -- 3.6. Approximating Polygons for the Peano Curve -- 3.7. Wunderlich's Versions of the Peano Curve -- 3.8. A Three-Dimensional Peano Curve -- 3.9. Problems -- 4. Sierpiński's Space-Filling Curve -- 4.1. Sierpiński's Original Definition -- 4.2. Geometric Generation and Knopp's Representation of the Sierpiński Curve -- 4.3. Representation of the Sierphiski-Knopp Curve in Terms of Quaternaries -- 4.4. Nowhere

Differentiability of the Sierpiński-Knopp Curve -- 4.5. Approximating Polygons for the Sierpiński-Knopp Curve -- 4.6. Pólya's Generalization of the Sierpiński-Knopp Curve -- 4.7. Problems -- 5. Lebesgue's Space-Filling Curve -- 5.1. The Cantor Set -- 5.2. Properties of the Cantor Set -- 5.3. The Cantor Function and the Devil's Staircase -- 5.4. Lebesgue's Definition of a Space-Filling Curve -- 5.5. Approximating Polygons for the Lebesgue Curve -- 5.6. Problems -- 6. Continuous Images of a Line Segment -- 6.1. Preliminary Remarks and a Global Characterization of Continuity -- 6.2. Compact Sets -- 6.3. Connected Sets -- 6.4. Proof of Netto's Theorem -- 6.5. Locally Connected Sets -- 6.6. A Theorem by Hausdorff -- 6.7. Pathwise Connectedness -- 6.8. The Hahn-Mazurkiewicz Theorem -- 6.9. Generation of Space-Filling Curves by Stochastically Independent Functions -- 6.10. Representation of a Space-Filling Curve by an Analytic Function -- 6.11. Problems -- 7. Schoenberg's Space-Filling Curve -- 7.1. Definition and Basic Properties -- 7.2. The Nowhere Differentiability of the Schoenberg Curve -- 7.3. Approximating Polygons -- 7.4. A Three-Dimensional Schoenberg Curve -- 7.5. An n -Dimensional Schoenberg Curve -- 7.6. Problems -- 8. Jordan Curves of Positive Lebesgue Measure -- 8.1. Jordan Curves -- 8.2. Osgood's Jordan Curves of Positive Measure -- 8.3. The Osgood Curves of Sierpiński and Knopp -- 8.4. Other Osgood Curves -- 8.5. Problems -- 9. Fractals -- 9.1. Examples -- 9.2. The Space where Fractals are Made -- 9.3. The Invariant Attractor Set -- 9.4. Similarity Dimension -- 9.5. Cantor Curves -- 9.6. The Heighway-Dragon -- 9.7. Problems -- A.1. Computer Programs 169 A.1.1. Computation of the Nodal Points of the Hilbert Curve -- A.1.2. Computation of the Nodal Points of the Peano Curve -- A.1.3. Computation of the Nodal Points of the Sierpiński-Knopp Curve -- A.1.4. Plotting Program for the Approximating Polygons of the Schoenberg Curve -- A.2. Theorems from Analysis -- A.2.1. Binary and Other Representations -- A.2.2. Condition for Non-Differentiability -- A.2.3. Completeness of the Euclidean Space -- A.2.4. Uniform Convergence -- A.2.5. Measure of the Intersection of a Decreasing Sequence of Sets -- A.2.6. Cantor's Intersection Theorem -- A.2.7. Infinite Products -- References.

Sommario/riassunto

The subject of space-filling curves has fascinated mathematicians for over a century and has intrigued many generations of students of mathematics. Working in this area is like skating on the edge of reason. Unfortunately, no comprehensive treatment has ever been attempted other than the gallant effort by W. Sierpiński in 1912. At that time, the subject was still in its infancy and the most interesting and perplexing results were still to come. Besides, Sierpiński's paper was written in Polish and published in a journal that is not readily accessible (Sierpiński [2]). Most of the early literature on the subject is in French, German, and Polish, providing an additional *raison d'être* for a comprehensive treatment in English. While there was, understandably, some intensive research activity on this subject around the turn of the century, contributions have, nevertheless, continued up to the present and there is no end in sight, indicating that the subject is still very much alive. The recent interest in fractals has refocused interest on space-filling curves, and the study of fractals has thrown some new light on this small but venerable part of mathematics. This monograph is neither a textbook nor an encyclopedic treatment of the subject nor a historical account, but it is a little of each. While it may lend structure to a seminar or pro-seminar, or be useful as a supplement in a course on topology or mathematical analysis, it is primarily intended for self-study by the aficionados of classical analysis.

