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Autore	Karris Steven T
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Nota di contenuto	Signals and Systems -- with MATLAB Computing -- and SimulinkO Modeling -- Fifth Edition -- Steven T. Karris -- Preface Signals and Systems Fifth -- Preface -- TOC Signals and Systems Fifth -- Chapter 01 Signals and Systems Fifth -- Chapter 02 Signals and Systems Fifth -- Chapter 2 -- The Laplace Transformation -- his chapter begins with an introduction to the Laplace transformation, definitions, and properties of the Laplace transformation. The initial value and final value theorems are also discussed and proved. It continues with the derivation of the Laplac... -- 2.1 Definition of the Laplace Transformation -- The two-sided or bilateral Laplace Transform pair is defined as -- (2.1) -- (2.2) -- where denotes the Laplace transform of the time function , denotes the Inverse Laplace transform, and is a complex variable whose real part is , and imaginary part , that is, . -- In most problems, we are concerned with values of time greater than some reference time, say , and since the initial conditions are generally known, the two-sided Laplace transform pair of (2.1) and (2.2) simplifies to the unilateral or one-sided Lap... -- (2.3) -- (2.4) -- The Laplace Transform of (2.3) has meaning only if the integral converges (reaches a limit), that is, if -- (2.5) -- To determine the conditions that will ensure us that the integral of (2.3) converges, we rewrite (2.5) as -- (2.6) -- The term in the integral of (2.6) has magnitude of unity, i.e., ,

and thus the condition for convergence becomes -- (2.7) --
 Fortunately, in most engineering applications the functions are of exponential order. Then, we can express (2.7) as, -- (2.8) -- and we see that the integral on the right side of the inequality sign in (2.8), converges if $\sigma > \sigma_0$. Therefore, we conclude that if $f(t)$ is of exponential order, the Laplace transform exists if -- (2.9) -- where σ_0 denotes the real part of the complex variable s .

Evaluation of the integral of (2.4) involves contour integration in the complex plane, and thus, it will not be attempted in this chapter. We will see in the next chapter that many Laplace transforms can be inverted with the use of a few standard pairs. -- In our subsequent discussion, we will denote transformation from the time domain to the complex frequency domain, and vice versa, as -- (2.10) --

2.2 Properties and Theorems of the Laplace Transform

The most common properties and theorems of the Laplace transform are presented in Subsections 2.2.1 through 2.2.13 below.

2.2.1 Linearity Property

The linearity property states that if $f_1(t)$ and $f_2(t)$ have Laplace transforms $F_1(s)$ and $F_2(s)$ respectively, and a and b are arbitrary constants, then, -- (2.11) --

Proof: -- Note 1: -- It is desirable to multiply by the unit step function to eliminate any unwanted non-zero values of $f(t)$ for $t < 0$. --

2.2.2 Time Shifting Property

The time shifting property states that a right shift in the time domain by units, corresponds to multiplication by $e^{-s\tau}$ in the complex frequency domain. Thus, -- (2.12) --

Proof: -- (2.13) --

Now, we let $u = t - \tau$, then, $du = dt$, and $t = u + \tau$. With these substitutions and with $f(u) = 0$ for $u < 0$, the second integral on the right side of (2.13) is expressed as --

2.2.3 Frequency Shifting Property

The frequency shifting property states that if we multiply a time domain function by an exponential function $e^{-\alpha t}$ where α is an arbitrary positive constant, this multiplication will produce a shift of the s variable in the complex frequency domain by α units... -- (2.14) --

Proof: -- Note 2: -- A change of scale is represented by multiplication of the time variable by a positive scaling factor a . Thus, the function after scaling the time axis, becomes $f(at)$. --

2.2.4 Scaling Property

Let a be an arbitrary positive constant -- then, the scaling property states that -- (2.15) --

Proof: -- and letting $u = at$, we obtain --

Note 3:.

Generally, the initial value of $f(t)$ is taken at $t = 0^-$ to include any discontinuity that may be present at $t = 0$. If it is known that no such discontinuity exists at $t = 0$, we simply interpret $f(0^-)$ as $f(0)$. --

2.2.5 Differentiation in Time Domain Property

The differentiation in time domain property states that differentiation in the time domain corresponds to multiplication by s in the complex frequency domain, minus the initial value of $f(t)$ at $t = 0^-$. Thus, -- (2.16) --

Proof: -- Using integration by parts where $u = f(t)$ and $dv = e^{-st} dt$ -- (2.17) -- we let $u = f(t)$ and $dv = e^{-st} dt$. Then, $du = f'(t) dt$, and $v = -\frac{1}{s} e^{-st}$, and thus --

The time differentiation property can be extended to show that -- (2.18) -- (2.19) -- and in general -- (2.20) --

-- To prove (2.18), we let $u = f(t)$ and as we found above, $dv = e^{-st} dt$. Then, --

Relations (2.19) and (2.20) can be proved by similar procedures. -- We must remember that the terms $f(0^-)$, and so on, represent the initial conditions. Therefore, when all initial conditions are zero, and we differentiate a time function n times, this corresponds to multiplied by s^n to the power. --

2.2.6 Differentiation in Complex Frequency Domain Property

This property states that differentiation in complex frequency domain and multiplication by $-t$ in the time domain, corresponds to multiplication of $F(s)$ by -1 in the time domain. In other words, -- (2.21) --

Proof: -- Differentiating with respect to s and applying Leibnitz's rule for differentiation under the integral, we obtain -- In general, -- (2.22) --

The proof for (2.22) follows by taking the second and higher-order derivatives of $f(t)$ with respect to t . --

2.2.7 Integration in Time Domain Property

This property states that integration in time domain corresponds to

divided by plus the initial value of $x(0^-)$ at $t = 0^-$, also divided by s . That is, -- (2.23) -- Proof: -- We begin by expressing the integral on the left side of (2.23) as two integrals, that is, -- (2.24). The first integral on the right side of (2.24), represents a constant value since neither the upper, nor the lower limits of integration are functions of time, and this constant is an initial condition denoted as $x(0^-)$. We will find the Laplace transform ... -- (2.25) -- This is the value of the first integral in (2.24). Next, we will show that -- We let ϵ -- then, $\lim_{\epsilon \rightarrow 0^+}$ -- and -- Now, -- (2.26) -- and the proof of (2.23) follows from (2.25) and (2.26). --

2.2.8 Integration in Complex Frequency Domain Property

This property states that integration in complex frequency domain with respect to s corresponds to division of a time function by the variable t , provided that the limit exists. Thus, -- (2.27) -- Proof: -- Integrating both sides from 0^- to ∞ , we obtain -- Next, we interchange the order of integration, i.e., -- and performing the inner integration on the right side integral with respect to s , we obtain --

2.2.9 Time Periodicity Property

The time periodicity property states that a periodic function of time with period T corresponds to the integral divided by T in the complex frequency domain. Thus, if we let $x(t)$ be a periodic function with period T , that is, $x(t) = x(t + T)$, for we obtain the transform pair -- (2.28) -- Proof: -- The Laplace transform of a periodic function can be expressed as -- In the first integral of the right side, we let $\tau = t - nT$, in the second, $\tau = t - (n-1)T$, in the third, $\tau = t - (n-2)T$, and so on. The areas under each period of $x(t)$ are equal, and thus the upper and lower limits of integration are the same for each integral. Then, -- (2.29) -- Since the function is periodic, i.e., $x(t) = x(t + T)$, we can write (2.29) as -- (2.30) -- By application of the binomial theorem, that is, -- (2.31) -- we find that expression (2.30) reduces to --

2.2.10 Initial Value Theorem

The initial value theorem states that the initial value of the time function can be found from its Laplace transform multiplied by s and letting $s \rightarrow \infty$. That is, -- (2.32) -- Proof: -- From the time domain differentiation property, -- or -- Taking the limit of both sides by letting $s \rightarrow \infty$, we obtain -- Interchanging the limiting process, we obtain -- and since -- the above expression reduces to -- or --

2.2.11 Final Value Theorem

The final value theorem states that the final value of the time function can be found from its Laplace transform multiplied by s , then, letting $s \rightarrow 0$. That is, -- (2.33) -- Proof: -- From the time domain differentiation property, -- or -- Taking the limit of both sides by letting $s \rightarrow 0$, we obtain -- and by interchanging the limiting process, the expression above is written as -- Also, since -- it reduces to -- Therefore, --

2.2.12 Convolution in Time Domain Property

Convolution in the time domain corresponds to multiplication in the complex frequency domain, that is, -- (2.34) -- Proof: -- (2.35) -- We let $\tau = t - \epsilon$ -- then, $\lim_{\epsilon \rightarrow 0^+}$ -- and $\lim_{\epsilon \rightarrow 0^+}$. Then, by substitution into (2.35), --

2.2.13 Convolution in Complex Frequency Domain Property

Convolution in the complex frequency domain divided by s , corresponds to multiplication in the time domain. That is, -- (2.36) -- Proof: -- (2.37) -- and recalling that the Inverse Laplace transform from (2.2) is -- by substitution into (2.37), we obtain -- We observe that the bracketed integral is -- therefore, -- For easy reference, the Laplace transform pairs and theorems are summarized in Table 2.1. --

2.3 Laplace Transforms of Common Functions of Time

In this section, we will derive the Laplace transform of common functions of time. They are presented in Subsections 2.3.1 through 2.3.11 below. --

2.3.1 Laplace Transform of the Unit Step Function

We begin with the definition of the Laplace transform, that is, -- or -- Thus, we have obtained the transform pair.

students, this text is an introduction to signal and system analysis, digital signal processing, and the design of analog and digital filters. The text also serves as a self-study guide for professionals who want to review the fundamentals. The expanded fifth edition contains additional information on window functions, the cross correlation and autocorrelation functions, a discussion on nonlinear systems including an example that derives its describing function, as well as additional end-of-chapter exercises.
