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		Set Theory
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		Forcing (Teoria de models)
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	Nota di contenuto	Part I: Preliminary -- 1 The Setting -- 2 First-Order Logic in a Nutshell -- 3 Axioms of Set Theory -- Part II: Topics in Combinatorial Set Theory -- 4 Overture: Ramsey's Theorem -- 5 Cardinal Relations in ZF Only -- 6 Forms of Choice -- 7 How to Make Two Balls from One -- 8 Models of Set Theory with Atoms -- 9 Thirteen Cardinals and Their Relations -- 10 The Shattering Number Revisited -- 11 Happy Families and Their Relatives -- 12 Coda: A Dual Form of Ramsey's Theorem -- Part III: From Martin's Axiom to Cohen's Forcing -- 13 The Idea of Forcing -- 14 Martin's Axiom -- 15 The Notion of Forcing -- 16 Proving Unprovability -- 17 Models in Which AC Fails -- 18 Combining Forcing Notions -- 19 Models in Which p=c -- 20 Suslin's Problem -- Part IV: Combinatorics of Forcing Extensions -- 21 Properties of Forcing Extensions -- 22 Cohen Forcing Revisited -- 23 Sacks Forcing -- 24 Silver-Like Forcing Notions -- 25 Miller Forcing -- 26 Mathias Forcing -- 27 Laver Forcing -- 28 How Many Ramsey Ultrafilters Exist?

-- 29 Suite.

Sommario/riassunto	This book, now in a revised and extended third edition, provides a comprehensive and accessible introduction to modern axiomatic set theory. After an overview of basic notions in combinatorics and first-order logic, and discussing in great detail the axioms of set theory, the author outlines in the second part the main topics of classical set theory, including Ramsey theory and the axiom of choice. As an application of the axiom of choice, a complete proof of Robinson's construction for doubling a ball by dividing it into only five parts is given. For the new edition, the chapter on permutation models has been extended, and recent results in set theory without the axiom of choice and about cardinal characteristics have been added. The third part explains the sophisticated technique of forcing from scratch, now including more details about iterated forcing. The technique is then used to show that certain statements are neither provable nor disprovable from the axioms of set theory. In particular, it is shown that both Martin's Axiom and Suslin's Hypothesis are independent of the axioms of set theory. The final part, with a new chapter on Laver forcing, is mainly concerned with consistency results obtained by iterations of forcing notions such as Cohen forcing, Sacks forcing, and Mathias forcing. The part begins with an extended chapter on countable support iterations of proper forcing notions, now also including proofs of some preservation theorems such as preservation of properness and of certain ultrafilters. In the following chapters, various consistency results concerning possible relations between cardinal characteristics and the existence of Ramsey ultrafilters are presented. For example, a detailed proof of Shelah's astonishing construction of a model with finitely many Ramsey ultrafilters is given. Written for graduate students in axiomatic set theory, Combinatorial Set Theory will appeal to all researchers interested in the foundations of mathematics. With extensive reference lists, historical remarks, and related results at the end of the chapters, this book is also suitable for self-study.