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	Soggetti	Logic, Symbolic and mathematical
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	Nota di contenuto	0. A Framework for Metamathematics -- Part I Introduction to First-Order Logic -- 1 Syntax: The Grammar of Symbols -- 2 The Art of Proof -- 3 Semantics: Making Sense of the Symbols -- Part II Gödel's Completeness Theorem -- 4 Maximally Consistent Extensions -- 5 The Completeness Theorem -- 6 Language Extensions by Definitions -- Part III Gödel's Incompleteness Theorems -- 7 Countable Models of Peano Arithmetic -- 8 Arithmetic in Peano Arithmetic -- 9 Gödelisation of Peano Arithmetic -- 10 The First Incompleteness Theorem -- 11 The Second Incompleteness Theorem -- 12 Completeness of Presburger Arithmetic -- Part IV The Axiom System ZFC -- 13 The Axioms of Set Theory (ZFC) -- 14 Models of Set Theory -- 15 Models and Ultraproducts -- 16 Models of Peano Arithmetic -- 17 Models of the Real Numbers -- Tautologies -- Solutions -- References -- Index.
	Sommario/riassunto	This book provides a concise and self-contained introduction to the foundations of mathematics. The first part covers the fundamental notions of mathematical logic, including logical axioms, formal proofs and the basics of model theory. Building on this, in the second and

third part of the book the authors present detailed proofs of Gödel's classical completeness and incompleteness theorems. In particular, the book includes a full proof of Gödel's second incompleteness theorem which states that it is impossible to prove the consistency of arithmetic within its axioms. The final part is dedicated to an introduction into modern axiomatic set theory based on Zermelo's axioms, containing also a presentation of Gödel's constructible universe of sets. A recurring theme in the whole book consists of standard and non-standard models of several theories, such as Peano arithmetic, Presburger arithmetic and the real numbers. In addition, the corrected, revised and extended second edition now provides detailed solutions to all exercises. The book addresses undergraduate mathematics students and is suitable for a one or two semester introductory course into logic and set theory.