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stability -- 5.1 Simple examples -- 5.2 Stability of equilibrium solutions -- 5.3 Stability of periodic solutions -- 5.4 Linearisation -- 5.5 Exercises -- 6 Linear Equations -- 6.1 Equations with constant coefficients -- 6.2 Equations with coefficients which have a limit -- 6.3 Equations with periodic coefficients -- 6.4 Exercises -- 7 Stability by linearisation -- 7.1 Asymptotic stability of the trivial solution -- 7.2 Instability of the trivial solution -- 7.3 Stability of periodic solutions of autonomous equations -- 7.4 Exercises -- 8 Stability analysis by the direct method -- 8.1 Introduction -- 8.2 Lyapunov functions -- 8.3 Hamiltonian systems and systems with first integrals -- 8.4 Applications and examples -- 8.5 Exercises -- 9 Introduction to perturbation theory -- 9.1 Background and elementary examples -- 9.2 Basic material -- 9.3 Naïve expansion -- 9.4 The Poincaré expansion theorem -- 9.5 Exercises -- 10 The Poincaré-Lindstedt method -- 10.1 Periodic solutions of autonomous second-order equations -- 10.2 Approximation of periodic solutions on arbitrary long time-scales -- 10.3 Periodic solutions of equations with forcing terms -- 10.4 The existence of periodic solutions -- 10.5 Exercises -- 11 The method of averaging -- 11.1 Introduction -- 11.2 The Lagrange standard form -- 11.3 Averaging in the periodic case -- 11.4 Averaging in the general case -- 11.5 Adiabatic invariants -- 11.6 Averaging over one angle, resonance manifolds -- 11.7 Averaging over more than one angle, an introduction -- 11.8 Periodic solutions -- 11.9 Exercises -- 12 Relaxation Oscillations -- 12.1 Introduction -- 12.2 Mechanical systems with large friction -- 12.3 The van der Pol-equation -- 12.4 The Volterra-Lotka equations -- 12.5 Exercises -- 13 Bifurcation Theory -- 13.1 Introduction -- 13.2 Normalisation -- 13.3 Averaging and normalisation -- 13.4 Centre manifolds -- 13.5 Bifurcation of equilibrium solutions and Hopf bifurcation -- 13.6 Exercises -- 14 Chaos -- 14.1 Introduction and historical context -- 14.2 The Lorenz-equations -- 14.3 Maps associated with the Lorenz-equations -- 14.4 One-dimensional dynamics -- 14.5 One-dimensional chaos: the quadratic map -- 14.6 One-dimensional chaos: the tent map -- 14.7 Fractal sets -- 14.8 Dynamical characterisations of fractal sets -- 14.9 Lyapunov exponents -- 14.10 Ideas and references to the literature -- 15 Hamiltonian systems -- 15.1 Introduction -- 15.2 A nonlinear example with two degrees of freedom -- 15.3 Birkhoff-normalisation -- 15.4 The phenomenon of recurrence -- 15.5 Periodic solutions -- 15.6 Invariant tori and chaos -- 15.7 The KAM theorem -- 15.8 Exercises -- Appendix 1: The Morse lemma -- Appendix 2: Linear periodic equations with a small parameter -- Appendix 3: Trigonometric formulas and averages -- Appendix 4: A sketch of Cotton's proof of the stable and unstable manifold theorem 3.3 -- Appendix 5: Bifurcations of self-excited oscillations -- Appendix 6: Normal forms of Hamiltonian systems near equilibria -- Answers and hints to the exercises -- References.

## Sommario/riassunto

On the subject of differential equations many elementary books have been written. This book bridges the gap between elementary courses and research literature. The basic concepts necessary to study differential equations - critical points and equilibrium, periodic solutions, invariant sets and invariant manifolds - are discussed first. Stability theory is then developed starting with linearisation methods going back to Lyapunov and Poincaré. In the last four chapters more advanced topics like relaxation oscillations, bifurcation theory, chaos in mappings and differential equations, Hamiltonian systems are introduced, leading up to the frontiers of current research: thus the reader can start to work on open research problems, after studying this book. This new edition contains an extensive analysis of fractal sets

with dynamical aspects like the correlation- and information dimension. In Hamiltonian systems, topics like Birkhoff normal forms and the Poincaré-Birkhoff theorem on periodic solutions have been added. There are now 6 appendices with new material on invariant manifolds, bifurcation of strongly nonlinear self-excited systems and normal forms of Hamiltonian systems. The subject material is presented from both the qualitative and the quantitative point of view, and is illustrated by many examples.

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