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## Sommario/riassunto

"The  $L_p$ -Brunn-Minkowski theory for  $p < 1$ , proposed by Firey and developed by Lutwak in the 90's, replaces the Minkowski addition of convex sets by its  $L_p$  counterpart, in which the support functions are added in  $L_p$ -norm. Recently, Boroczky, Lutwak, Yang and Zhang have proposed to extend this theory further to encompass the range. In particular, they conjectured an  $L_p$ -Brunn-Minkowski inequality for origin-symmetric convex bodies in that range, which constitutes a strengthening of the classical Brunn-Minkowski inequality. Our main result confirms this conjecture locally for all (smooth) origin-symmetric convex bodies in  $R^n$  and. In addition, we confirm the local log-Brunn-Minkowski conjecture (the case  $p = 2$ ) for small-enough  $C^2$ -perturbations of the unit-ball of  $l_q$  for  $q \geq 2$ , when the dimension  $n$  is sufficiently large, as well as for the cube, which we show is the conjectural extremal case. For unit-balls of  $l_q$  with  $q \leq 2$ , we confirm an analogous result for  $p$ , a universal constant. It turns out that the local version of these conjectures is equivalent to a minimization problem for a spectral-gap parameter associated with a certain differential operator, introduced by Hilbert (under different normalization) in his proof of the Brunn-Minkowski inequality. As applications, we obtain local uniqueness results in the even  $L_p$ -Minkowski problem, as well as improved stability estimates in the Brunn-Minkowski and anisotropic isoperimetric inequalities"--