1. Record Nr. UNINA9910962003503321

Autore	Mac Lane Saunders <1909-2005>
Titolo	Sheaves in Geometry and Logic : A First Introduction to Topos Theory / / by Saunders MacLane, Ieke Moerdijk
Pubbl/distr/stampa	New York, NY : , : Springer New York : , : Imprint : Springer, , 1994
ISBN	9781461209270 1461209277
Edizione	[1st ed. 1994.]
Descrizione fisica	1 online resource (XII, 630 p.)
Collana	Universitext, , 2191-6675
Disciplina	512/.55
Soggetti	Geometry K-theory Logic, Symbolic and mathematical K-Theory Mathematical Logic and Foundations
Lingua di pubblicazione	Inglese
Formato	Materiale a stampa
Livello bibliografico	Monografia
Note generali	Bibliographic Level Mode of Issuance: Monograph
Nota di bibliografia	Includes bibliographical references and indexes.
Nota di contenuto	Prologue -- Categorial Preliminaries -- I. Categories of Functors -- 1. The Categories at Issue -- 2. Pullbacks -- 3. Characteristic Functions of Subobjects -- 4. Typical Subobject Classifiers -- 5. Colimits -- 6. Exponentials -- 7. Propositional Calculus -- 8. Heyting Algebras -- 9. Quantifiers as Adjoints -- Exercises -- II. Sheaves of Sets -- 1. Sheaves -- 2. Sieves and Sheaves -- 3. Sheaves and Manifolds -- 4. Bundles -- 5. Sheaves and Cross-Sections -- 6. Sheaves as Étale Spaces -- 7. Sheaves with Algebraic Structure -- 8. Sheaves are Typical -- 9. Inverse Image Sheaf -- Exercises -- III. Grothendieck Topologies and Sheaves -- 1. Generalized Neighborhoods -- 2. Grothendieck Topologies -- 3. The Zariski Site -- 4. Sheaves on a Site -- 5. The Associated Sheaf Functor -- 6. First Properties of the Category of Sheaves -- 7. Subobject Classifiers for Sites -- 8. Subsheaves -- 9. Continuous Group Actions -- Exercises -- IV. First Properties of Elementary Topoi -- 1. Definition of a Topos -- 2. The Construction of Exponentials -- 3. Direct Image -- 4. Monads and Beck's Theorem -- 5. The Construction of Colimits -- 6. Factorization and Images -- 7. The Slice Category as a Topos -- 8. Lattice and Heyting Algebra Objects in a Topos -- 9. The

Beck-Chevalley Condition -- 10. Injective Objects -- Exercises -- V. Basic Constructions of Topoi -- 1. Lawvere-Tierney Topologies -- 2. Sheaves -- 3. The Associated Sheaf Functor -- 4. Lawvere-Tierney Subsumes Grothendieck -- 5. Internal Versus External -- 6. Group Actions -- 7. Category Actions -- 8. The Topos of Coalgebras -- 9. The Filter-Quotient Construction -- Exercises -- VI. Topoi and Logic -- 1. The Topos of Sets -- 2. The Cohen Topos -- 3. The Preservation of Cardinal Inequalities -- 4. The Axiom of Choice -- 5. The Mitchell-Bénabou Language -- 6. Kripke-Joyal Semantics -- 7. Sheaf Semantics -- 8. Real Numbers in a Topos -- 9. Brouwer's Theorem: All Functions are Continuous -- 10. Topos-Theoretic and Set-Theoretic Foundations -- Exercises -- VII. Geometric Morphisms -- 1. Geometric Morphismsand Basic Examples -- 2. Tensor Products -- 3. Group Actions -- 4. Embeddings and Surjections -- 5. Points -- 6. Filtering Functors -- 7. Morphisms into Grothendieck Topoi -- 8. Filtering Functors into a Topos -- 9. Geometric Morphisms as Filtering Functors -- 10. Morphisms Between Sites -- Exercises -- VIII. Classifying Topoi -- 1. Classifying Spaces in Topology -- 2. Torsors -- 3. Classifying Topoi -- 4. The Object Classifier -- 5. The Classifying Topos for Rings -- 6. The Zariski Topos Classifies Local Rings -- 7. Simplicial Sets -- 8. Simplicial Sets Classify Linear Orders -- Exercises -- IX. Localic Topoi -- 1. Locales -- 2. Points and Sober Spaces -- 3. Spaces from Locales -- 4. Embeddings and Surjections of Locales -- 5. Localic Topoi -- 6. Open Geometric Morphisms -- 7. Open Maps of Locales -- 8. Open Maps and Sites -- 9. The Diaconescu Cover and Barr's Theorem -- 10. The Stone Space of a Complete Boolean Algebra -- 11. Deligne's Theorem -- Exercises -- X. Geometric Logic and Classifying Topoi -- 1. First-OrderTheories -- 2. Models in Topoi -- 3. Geometric Theories -- 4. Categories of Definable Objects -- 5. Syntactic Sites -- 6. The Classifying Topos of a Geometric Theory -- 7. Universal Models -- Exercises -- Appendix: Sites for Topoi -- Epilogue -- Index of Notation.

Sommario/riassunto

We dedicate this book to the memory of J. Frank Adams. His clear insights have inspired many mathematicians, including both of us. In January 1989, when the first draft of our book had been completed, we heard the sad news of his untimely death. This has cast a shadow on our subsequent work. Our views of topos theory, as presented here, have been shaped by continued study, by conferences, and by many personal contacts with friends and colleagues-including especially O. Bruno, P. Freyd, J.M.E. Hyland, P.T. Johnstone, A. Joyal, A. Kock, F.W. Lawvere, G.E. Reyes, R Solovay, R Swan, RW. Thomason, M. Tierney, and G.C. Wraith. Our presentation combines ideas and results from these people and from many others, but we have not endeavored to specify the various original sources. Moreover, a number of people have assisted in our work by pro- viding helpful comments on portions of the manuscript. In this respect, we extend our hearty thanks in particular to P. Corazza, K. Edwards, J. Greenlees, G. Janelidze, G. Lewis, and S. Schanuel.