

1. Record Nr.	UNINA9910866583403321
Autore	Liu Xuefeng <1961->
Titolo	Guaranteed Computational Methods for Self-Adjoint Differential Eigenvalue Problems // by Xuefeng Liu
Pubbl/distr/stampa	Singapore : , : Springer Nature Singapore : , : Imprint : Springer, , 2024
ISBN	9789819735778 9789819735761
Edizione	[1st ed. 2024.]
Descrizione fisica	1 online resource (139 pages)
Collana	SpringerBriefs in Mathematics, , 2191-8201
Disciplina	512.9436
Soggetti	Mathematical analysis Functional analysis Mathematics - Data processing Analysis Functional Analysis Computational Mathematics and Numerical Analysis Espais vectorials Anàlisi funcional Llibres electrònics
Lingua di pubblicazione	Inglese
Formato	Materiale a stampa
Livello bibliografico	Monografia
Nota di bibliografia	Includes bibliographical references.
Nota di contenuto	Intro -- Preface -- Contents -- 1 Introduction to Eigenvalue Problems -- 1.1 Overview of Research on Rigorous Eigenvalue Bounds -- 1.2 Model Eigenvalue Problems -- 1.3 Sobolev Space Settings and Weak Formulation of Eigenvalue Problem -- 1.4 Min-Max Principle and Upper Eigenvalue Bounds -- 1.5 Finite Element Method -- 1.5.1 Mesh for Numerical Examples -- 1.5.2 Approximate Eigenvalue Evaluation -- 2 Explicit Error Estimation for Boundary Value Problems -- 2.1 Poisson's Equation and Its FE Solution -- 2.1.1 Poisson's Equation -- 2.1.2 Finite Element Solution -- 2.2 Interpolation Error Estimation and Several Constants -- 2.2.1 Interpolation Function and Error Estimation -- 2.2.2 Constants in the Trace Theorem -- 2.3 A Priori Error Estimate for Solutions with H2-Regularity -- 2.4 Error Estimate for Solutions Without H2-Regularity -- 2.4.1 Space Settings and Hypercircle in the

Prager-Syngé Theorem -- 2.4.2 A Posteriori Error Estimation -- 2.4.3 A Priori Error Estimation -- 2.4.4 Numerical Examples -- 2.4.4.1 Square Domain -- 2.4.4.2 L-Shaped Domain -- 2.5 Poisson's Equation with General Settings -- 2.5.1 General Hypercircle Involving $c(x)$ -- 2.5.2 A Priori Error Estimation -- 2.5.3 Computation of h and Its Upper Bound -- 2.5.4 Numerical Computation -- 2.6 Error Estimation for Stokes Equations -- 2.6.1 Problem Settings -- 2.6.2 Finite Element Spaces -- 2.6.2.1 Construction of V_h -- 2.6.2.2 Projection Operators -- 2.6.3 Explicit Error Estimation for FE Solutions -- 2.6.3.1 A Posteriori Error Estimation -- 2.6.3.2 A Priori Error Estimation -- 2.6.3.3 Computation of h -- 2.6.4 Numerical Computation Results -- 2.6.4.1 A Priori Error Estimation over 3D Domains -- 3 Fundamental Theorem for Explicit Eigenvalue Bounds -- 3.1 Eigenvalue Problem with Positive Definite $a(\cdot, \cdot)$ -- 3.1.1 Explicit Eigenvalue Bounds. 3.2 Eigenvalue Problems with Positive Semi-definite $a(\cdot, \cdot)$ -- 3.2.1 Problem Setting and Explicit Eigenvalue Bounds -- 3.3 Evaluation of the Constant in Projection Error Estimation -- 4 Explicit Eigenvalue Bounds for Various Differential Operators -- 4.1 Preparation: Non-conforming FEMs -- 4.1.1 Crouzeix-Raviart FEM -- 4.1.1.1 Interpolation Operator h_{CR} -- 4.1.1.2 Interpolation Error Constant $CCR(K)$ -- 4.1.2 Enriched Crouzeix-Raviart FEM -- 4.1.3 Composite Enriched Crouzeix-Raviart FEM -- 4.1.4 Fujino-Morley FEM -- 4.2 Laplacian Eigenvalue Problems -- 4.2.1 Case of $c=0$ ($-u = u$) -- 4.2.2 Case of $c > 0$ ($-u + cu = u$) -- 4.3 Stokes Eigenvalue Problems -- 4.3.1 Weak Formulation of the Stokes Eigenvalue Problem -- 4.3.2 Lower Bounds Using Non-conforming FEMs -- 4.3.3 Lower and Upper Bounds Using Conforming FEMs -- 4.3.4 Numerical Results -- 4.4 Steklov Eigenvalue Problems -- 4.4.1 Lower Bound Using Conforming FEMs -- 4.4.2 Lower Bound Using Non-conforming FEMs -- 4.4.3 Computation Results -- 4.5 Biharmonic Eigenvalue Problems -- 4.5.1 Lower Bounds Using Fujino-Morley FEMs -- 4.5.2 Computation Examples -- 5 Lehmann-Goerisch Method for High-Precision Eigenvalue Bounds -- 5.1 Lehmann-Goerisch Method -- 5.2 Application of the Lehmann-Goerisch Method -- 5.2.1 Dirichlet Eigenvalue Problems -- 5.2.2 Steklov Eigenvalue Problems -- 5.3 Computational Results and Applications -- 5.3.1 Eigenvalue Bounds for Dirichlet Eigenvalues -- 5.3.2 Eigenvalue Bounds for Steklov Eigenvalues -- 6 Guaranteed Eigenfunction Computation -- 6.1 Preliminaries -- 6.1.1 Distance Between Subspaces -- 6.1.2 Eigenspaces for Operators -- 6.2 Algorithm I: Rayleigh Quotient-Based Error Estimation -- 6.3 Algorithm II: Residual-Based Estimation -- 6.3.1 Extension of the Davis-Kahan sin Theorem to Weakly Formulated Problems -- 6.3.2 Weakly Formulated Residual Error Estimation. 6.3.3 Direct Estimate of a : Another Application of the Davis-Kahan Theorem -- 6.4 Algorithm III: Galerkin Projection-Based Estimation -- 6.4.1 A Priori Error Estimation for FE Solutions of Boundary Value Problems -- 6.4.2 Galerkin Projection-Based Estimate in L_2 Norm -- 6.5 Numerical Examples -- 6.5.1 Unit Square Domain -- 6.5.2 L-Shaped Domain -- A Introduction to VFEM Library -- References.

Sommario/riassunto

This monograph presents a study of newly developed guaranteed computational methodologies for eigenvalue problems of self-adjoint differential operators. It focuses on deriving explicit lower and upper bounds for eigenvalues, as well as explicit estimations for eigenfunction approximations. Such explicit error estimations rely on the finite element method (FEM) along with a new theory of explicit quantitative error estimation, diverging from traditional studies that primarily focus on qualitative results. To achieve quantitative error estimation, the monograph begins with an extensive analysis of the

hypercircle method, that is, the Prager–Synge theorem. It introduces a novel a priori error estimation technique based on the hypercircle method. This facilitates the explicit estimation of Galerkin projection errors for equations such as Poisson's and Stokes', which are crucial for obtaining lower eigenvalue bounds via conforming FEMs. A thorough exploration of the fundamental theory of projection-based explicit lower eigenvalue bounds under a general setting of eigenvalue problems is also offered. This theory is extensively detailed when applied to model eigenvalue problems associated with the Laplace, biharmonic, Stokes, and Steklov differential operators, which are solved by either conforming or non-conforming FEMs. Moreover, there is a detailed discussion on the Lehmann–Goerisch theorem for the purpose of high-precision eigenvalue bounds, showing its relationship with previously established theorems, such as Lehmann–Maehly's method and Kato's bound. The implementation details of this theorem with FEMs, a topic rarely covered in existing literature, are also clarified. Lastly, the monograph introduces three new algorithms to estimate eigenfunction approximation errors, revealing the potency of classical theorems. Algorithm I extends Birkhoff's result that works for simple eigenvalues to handle clustered eigenvalues, while Algorithm II generalizes the Davis–Kahan theorem, initially designed for strongly formulated eigenvalue problems, to address weakly formulated eigenvalue problems. Algorithm III utilizes the explicit Galerkin projection error estimation to efficiently handle Galerkin projection-based approximations.
