1. Record Nr. UNINA9910815403803321 Autore Newman Stephen C. <1952-> Titolo A classical introduction to Galois theory / / Stephen C. Newman Pubbl/distr/stampa Hoboken, N.J., : Wiley, c2012 **ISBN** 1-280-67898-4 9786613655912 1-118-33684-4 1-118-33681-X 1-118-33667-4 Edizione [1st ed.] Descrizione fisica 1 online resource (298 p.) Classificazione MAT003000 Disciplina 512/.32 Soggetti Galois theory Lingua di pubblicazione Inglese **Formato** Materiale a stampa Livello bibliografico Monografia Description based upon print version of record. Note generali Nota di bibliografia Includes bibliographical references and index. A CLASSICAL INTRODUCTION TO GALOIS THEORY: CONTENTS: Nota di contenuto PREFACE; 1 CLASSICAL FORMULAS; 1.1 Quadratic Polynomials; 1.2 Cubic Polynomials; 1.3 Quartic Polynomials; 2 POLYNOMIALS AND FIELD THEORY: 2.1 Divisibility: 2.2 Algebraic Extensions: 2.3 Degree of Extensions; 2.4 Derivatives; 2.5 Primitive Element Theorem; 2.6 Isomorphism Extension Theorem and Splitting Fields; 3 FUNDAMENTAL THEOREM ON SYMMETRIC POLYNOMIALS AND DISCRIMINANTS; 3.1 Fundamental Theorem on Symmetric Polynomials; 3.2 Fundamental Theorem on Symmetric Rational Functions; 3.3 Some Identities Based on Elementary Symmetric Polynomials 3.4 Discriminants 3.5 Discriminants and Subfields of the Real Numbers: 4 IRREDUCIBILITY AND FACTORIZATION; 4.1 Irreducibility Over the Rational Numbers; 4.2 Irreducibility and Splitting Fields; 4.3 Factorization and Adjunction: 5 ROOTS OF UNITY AND CYCLOTOMIC POLYNOMIALS; 5.1 Roots of Unity; 5.2 Cyclotomic Polynomials; 6 RADICAL EXTENSIONS AND SOLVABILITY BY RADICALS; 6.1 Basic Results on Radical Extensions; 6.2 Gauss's Theorem on Cyclotomic Polynomials; 6.3 Abel's Theorem on Radical Extensions; 6.4 Polynomials of Prime Degree; 7 GENERAL POLYNOMIALS AND THE

BEGINNINGS OF GALOIS THEORY

7.1 General Polynomials7.2 The Beginnings of Galois Theory; 8 CLASSICAL GALOIS THEORY ACCORDING TO GALOIS; 9 MODERN GALOIS THEORY; 9.1 Galois Theory and Finite Extensions; 9.2 Galois Theory and Splitting Fields; 10 CYCLIC EXTENSIONS AND CYCLOTOMIC FIELDS; 10.1 Cyclic Extensions; 10.2 Cyclotomic Fields; 11 GALOIS'S CRITERION FOR SOLVABILITY OF POLYNOMIALS BY RADICALS; 12 POLYNOMIALS OF PRIME DEGREE; 13 PERIODS OF ROOTS OF UNITY; 14 DENESTING RADICALS; 15 CLASSICAL FORMULAS REVISITED; 15.1 General Quadratic Polynomial; 15.2 General Cubic Polynomial; 15.3 General Quartic Polynomial

APPENDIX A COSETS AND GROUP ACTIONSAPPENDIX B CYCLIC GROUPS; APPENDIX C SOLVABLE GROUPS; APPENDIX D PERMUTATION GROUPS; APPENDIX E FINITE FIELDS AND NUMBER THEORY; APPENDIX F FURTHER READING; REFERENCES; INDEX

Sommario/riassunto

"This book provides an introduction to Galois theory and focuses on one central theme - the solvability of polynomials by radicals. Both classical and modern approaches to the subject are described in turn in order to have the former (which is relatively concrete and computational) provide motivation for the latter (which can be quite abstract). The theme of the book is historically the reason that Galois theory was created, and it continues to provide a platform for exploring both classical and modern concepts. This book examines a number of problems arising in the area of classical mathematics, and a fundamental question to be considered is: For a given polynomial equation (over a given field), does a solution in terms of radicals exist? That the need to investigate the very existence of a solution is perhaps surprising and invites an overview of the history of mathematics. The classical material within the book includes theorems on polynomials, fields, and groups due to such luminaries as Gauss, Kronecker, Lagrange, Ruffini and, of course, Galois. These results figured prominently in earlier expositions of Galois theory, but seem to have gone out of fashion. This is unfortunate since, aside from being of intrinsic mathematical interest, such material provides powerful motivation for the more modern treatment of Galois theory presented later in the book. Over the course of the book, three versions of the Impossibility Theorem are presented: the first relies entirely on polynomials and fields, the second incorporates a limited amount of group theory, and the third takes full advantage of modern Galois theory. This progression through methods that involve more and more group theory characterizes the first part of the book. The latter part of the book is devoted to topics that illustrate the power of Galois theory as a computational tool, but once again in the context of solvability of polynomial equations by radicals"--