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**Sommario/riassunto**

Aimed at researchers in mathematics and physics, this monograph, in which the author overviews the basic ideas in General Relativity, introduces the necessary mathematics and discusses some of the key open questions in the field.

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