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Autore	MacLane Saunders
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Nota di contenuto	Prologue -- Categorial Preliminaries -- I. Categories of Functors -- 1. The Categories at Issue -- 2. Pullbacks -- 3. Characteristic Functions of Subobjects -- 4. Typical Subobject Classifiers -- 5. Colimits -- 6. Exponentials -- 7. Propositional Calculus -- 8. Heyting Algebras -- 9. Quantifiers as Adjoints -- Exercises -- II. Sheaves of Sets -- 1. Sheaves -- 2. Sieves and Sheaves -- 3. Sheaves and Manifolds -- 4. Bundles -- 5. Sheaves and Cross-Sections -- 6. Sheaves as Étale Spaces -- 7. Sheaves with Algebraic Structure -- 8. Sheaves are Typical -- 9. Inverse Image Sheaf -- Exercises -- III. Grothendieck Topologies and Sheaves -- 1. Generalized Neighborhoods -- 2. Grothendieck Topologies -- 3. The Zariski Site -- 4. Sheaves on a Site -- 5. The Associated Sheaf Functor -- 6. First Properties of the Category of Sheaves -- 7. Subobject Classifiers for Sites -- 8. Subsheaves -- 9. Continuous Group Actions -- Exercises -- IV. First Properties of Elementary Topoi -- 1. Definition of a Topos -- 2. The Construction of Exponentials -- 3. Direct Image -- 4. Monads and Beck’s Theorem -- 5. The Construction of Colimits -- 6. Factorization and Images -- 7. The Slice Category as a Topos -- 8. Lattice and Heyting Algebra Objects in a Topos -- 9. The Beck-Chevalley Condition -- 10. Injective Objects -- Exercises -- V.

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Sommario/riassunto

We dedicate this book to the memory of J. Frank Adams. His clear insights have inspired many mathematicians, including both of us. In January 1989, when the first draft of our book had been completed, we heard the sad news of his untimely death. This has cast a shadow on our subsequent work. Our views of topos theory, as presented here, have been shaped by continued study, by conferences, and by many personal contacts with friends and colleagues-including especially O. Bruno, P. Freyd, J.M.E. Hyland, P.T. Johnstone, A. Joyal, A. Kock, F.W. Lawvere, G.E. Reyes, R Solovay, R Swan, RW. Thomason, M. Tierney, and G.C. Wraith. Our presentation combines ideas and results from these people and from many others, but we have not endeavored to specify the various original sources. Moreover, a number of people have assisted in our work by pro- viding helpful comments on portions of the manuscript. In this respect, we extend our hearty thanks in particular to P. Corazza, K. Edwards, J. Greenlees, G. Janelidze, G. Lewis, and S. Schanuel.