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theorem; 4.9 The groups  $SO(3)$  and  $SO(4)$ ; 4.10 Complex quadratic forms; 4.11 Complex inner-product spaces; 5: Clifford algebras; 5.1 Clifford algebras; 5.2 Existence; 5.3 Three involutions  
5.4 Centralizers, and the centre; 5.5 Simplicity; 5.6 The trace and quadratic form on  $A(E, q)$ ; 5.7 The group  $G(E, q)$  of invertible elements of  $A(E, q)$ ; 6: Classifying Clifford algebras; 6.1 Frobenius' theorem; 6.2 Clifford algebras  $A(E, q)$  with  $\dim E = 2$ ; 6.3 Clifford's theorem; 6.4 Classifying even Clifford algebras; 6.5 Cartan's periodicity law; 6.6 Classifying complex Clifford algebras; 7: Representing Clifford algebras; 7.1 Spinors; 7.2 The Clifford algebras  $A_{k,k}$ ; 7.3 The algebras  $B_{k,k+1}$  and  $A_{k,k+1}$ ; 7.4 The algebras  $A_{k+1,k}$  and  $A_{k+2,k}$ ; 7.5 Clifford algebras  $A(E, q)$  with  $\dim E = 3$   
7.6 Clifford algebras  $A(E, q)$  with  $\dim E = 4$ ; 7.7 Clifford algebras  $A(E, q)$  with  $\dim E = 5$ ; 7.8 The algebras  $A_6, B_7, A_7$  and  $A_8$ ; 8: Spin; 8.1 Clifford groups; 8.2 Pin and Spin groups; 8.3 Replacing  $q$  by  $\epsilon q$ ; 8.4 The spin group for odd dimensions; 8.5 Spin groups, for  $d = 2$ ; 8.6 Spin groups, for  $d = 3$ ; 8.7 Spin groups, for  $d = 4$ ; 8.8 The group  $Spin_5$ ; 8.9 Examples of spin groups for  $d \geq 6$ ; 8.10 Table of results; PART THREE: SOME APPLICATIONS; 9: Some applications to physics; 9.1 Particles with spin  $1/2$ ; 9.2 The Dirac operator; 9.3 Maxwell's equations; 9.4 The Dirac equation  
10: Clifford analyticity; 10.1 Clifford analyticity; 10.2 Cauchy's integral formula; 10.3 Poisson kernels and the Dirichlet problem; 10.4 The Hilbert transform; 10.5 Augmented Dirac operators; 10.6 Subharmonicity properties; 10.7 The Riesz transform; 10.8 The Dirac operator on a Riemannian manifold; 11: Representations of Spin and  $SO(d)$ ; 11.1 Compact Lie groups and their representations; 11.2 Representations of  $SU(2)$ ; 11.3 Representations of Spin and  $SO(d)$  for  $d \leq 4$ ; 12: Some suggestions for further reading; The algebraic environment; Quadratic spaces; Clifford algebras  
Clifford algebras and harmonic analysis

## Sommario/riassunto

Clifford algebras, built up from quadratic spaces, have applications in many areas of mathematics, as natural generalizations of complex numbers and the quaternions. They are famously used in proofs of the Atiyah-Singer index theorem, to provide double covers (spin groups) of the classical groups and to generalize the Hilbert transform. They also have their place in physics, setting the scene for Maxwell's equations in electromagnetic theory, for the spin of elementary particles and for the Dirac equation. This straightforward introduction to Clifford algebras makes the necessary algebraic background - including multilinear algebra, quadratic spaces and finite-dimensional real algebras - easily accessible to research students and final-year undergraduates. The author also introduces many applications in mathematics and physics, equipping the reader with Clifford algebras as a working tool in a variety of contexts.