

1. Record Nr.	UNINA9910789341503321
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Titolo	Space-Filling Curves [[electronic resource] /] / by Hans Sagan
Pubbl/distr/stampa	New York, NY : , : Springer New York : , : Imprint : Springer, , 1994
ISBN	1-4612-0871-8
Edizione	[1st ed. 1994.]
Descrizione fisica	1 online resource (XV, 194 p.)
Collana	Universitext, , 0172-5939
Classificazione	54F50 28A75 54-03 01A55 01A60
Disciplina	516.3/62
Soggetti	Geometry
Lingua di pubblicazione	Inglese
Formato	Materiale a stampa
Livello bibliografico	Monografia
Note generali	Bibliographic Level Mode of Issuance: Monograph
Nota di bibliografia	Includes bibliographical references and index.
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## Sommario/riassunto

The subject of space-filling curves has fascinated mathematicians for over a century and has intrigued many generations of students of mathematics. Working in this area is like skating on the edge of reason. Unfortunately, no comprehensive treatment has ever been attempted other than the gallant effort by W. Sierpiński in 1912. At that time, the subject was still in its infancy and the most interesting and perplexing results were still to come. Besides, Sierpiński's paper was written in Polish and published in a journal that is not readily accessible (Sierpiński [2]). Most of the early literature on the subject is in French, German, and Polish, providing an additional *raison d'être* for a comprehensive treatment in English. While there was, understandably, some intensive research activity on this subject around the turn of the century, contributions have, nevertheless, continued up to the present and there is no end in sight, indicating that the subject is still very much alive. The recent interest in fractals has refocused interest on space-filling curves, and the study of fractals has thrown some new light on this small but venerable part of mathematics. This monograph is neither a textbook nor an encyclopedic treatment of the subject nor a historical account, but it is a little of each. While it may lend structure to a seminar or pro-seminar, or be useful as a supplement in a course on topology or mathematical analysis, it is primarily intended for self-study by the aficionados of classical analysis.

