

the waveform of Figure 1.3. -- Figure 1.3. Waveform for -- In the waveform of Figure 1.3, the unit step function changes abruptly from to at . But if it changes at instead, it is denoted as . In this case, its waveform and definition are as shown in Figure 1.4 and relation (1.5) respectively. -- Figure 1.4. Waveform for -- (1.5). If the unit step function changes abruptly from to at , it is denoted as . In this case, its waveform and definition are as shown in Figure 1.5 and relation (1.6) respectively. -- Figure 1.5. Waveform for -- (1.6) -- Example 1.1 -- Consider the network of Figure 1.6, where the switch is closed at time . -- Figure 1.6. Network for Example 1.1 -- Express the output voltage as a function of the unit step function, and sketch the appropriate waveform. -- Solution: -- For this example, the output voltage for , and for . Therefore, -- (1.7) -- and the waveform is shown in Figure 1.7. -- Figure 1.7. Waveform for Example 1.1 -- Other forms of the unit step function are shown in Figure 1.8. -- Figure 1.8. Other forms of the unit step function -- Unit step functions can be used to represent other time-varying functions such as the rectangular pulse shown in Figure 1.9. -- Figure 1.9. A rectangular pulse expressed as the sum of two unit step functions -- Thus, the pulse of Figure 1.9(a) is the sum of the unit step functions of Figures 1.9(b) and 1.9(c) and it is represented as . -- The unit step function offers a convenient method of describing the sudden application of a volt age or current source. For example, a constant voltage source of applied at , can be denoted as . Likewise, a sinusoidal voltage source that is a... -- Example 1.2 -- Express the square waveform of Figure 1.10 as a sum of unit step functions. The vertical dotted lines indicate the discontinuities at , and so on. -- Figure 1.10. Square waveform for Example 1.2 -- Solution: -- Line segment has height , starts at , and terminates at . Then, as in Example 1.1, this segment is expressed as -- (1.8) -- Line segment has height , starts at and terminates at . This segment is expressed as -- (1.9) -- Line segment has height , starts at and terminates at . This segment is expressed as -- (1.10). Line segment has height , starts at , and terminates at . It is expressed as -- (1.11) -- Thus, the square waveform of Figure 1.10 can be expressed as the summation of  $(1.8)$  through  $(1.11)$ , that is,  $-(1.12)$ -- Combining like terms, we obtain -- (1.13) -- Example 1.3 -- Express the symmetric rectangular pulse of Figure 1.11 as a sum of unit step functions. -- Figure 1.11. Symmetric rectangular pulse for Example 1.3 -- Solution: -- This pulse has height , starts at , and terminates at . Therefore, with refer ence to Figures 1.5 and 1.8 (b), we obtain -- (1.14) -- Example 1.4 -- Express the symmetric triangular waveform of Figure 1.12 as a sum of unit step functions. -- Figure 1.12. Symmetric triangular waveform for Example 1.4 -- Solution: -- We first derive the equations for the linear segments and shown in Figure 1.13. -- Figure 1.13. Equations for the linear segments of Figure 1.12 -- For line segment , -- (1.15) -- and for line segment , -- (1.16) -- Combining (1.15) and (1.16), we obtain -- (1.17) -- Example 1.5 -- Express the waveform of Figure 1.14 as a sum of unit step functions. -- Figure 1.14. Waveform for Example 1.5 -- Solution: -- As in the previous example, we first find the equations of the linear segments linear segments and shown in Figure 1.15. -- Figure 1.15. Equations for the linear segments of Figure 1.14 -- Following the same procedure as in the previous examples, we obtain -- Multiplying the values in parentheses by the values in the brackets, we obtain -- and combining terms inside the brackets, we obtain -- (1.18) -- Two other functions of interest are the unit ramp function, and the unit impulse or delta function. We will introduce them with the examples that follow. -- Example 1.6.

In the network of Figure 1.16 is a constant current source and the switch is closed at time . Express the capacitor voltage as a function of the unit step. -- Figure 1.16. Network for Example 1.6 -- Solution: -- The current through the capacitor is , and the capacitor voltage is -- (1.19) -- where is a dummy variable. -- Since the switch closes at , we can express the current as -- (1.20) -- and assuming that for , we can write (1.19) as -- (1.21) -- or -- (1.22) -- Therefore, we see that when a capacitor is charged with a constant current, the voltage across it is a linear function and forms a ramp with slope as shown in Figure 1.17. -- Figure 1.17. Voltage across a capacitor when charged with a constant current source -- 1.3 The Unit Ramp Function -- The unit ramp function, denoted as , is defined as -- (1.23) -- where is a dummy variable. -- We can evaluate the integral of (1.23) by considering the area under the unit step function from as shown in Figure 1.18. -- Figure 1.18. Area under the unit step function from -- Therefore, we define as -- (1.24) -- Since is the integral of, then must be the derivative of , i.e., -- (1.25) -- Higher order functions of can be generated by repeated integration of the unit step function. For example, integrating twice and multiplying by , we define as -- (1.26) -- Similarly, -- (1.27) -- and in general, -- (1.28) -- Also, -- (1.29) -- Example 1.7 -- In the network of Figure 1.19, the switch is closed at time and for . Express the inductor voltage in terms of the unit step function. -- Figure 1.19. Network for Example 1.7 -- Solution: -- The voltage across the inductor is -- (1.30) -- and since the switch closes at , -- (1.31) -- Therefore, we can write (1.30) as -- (1.32). But, as we know, is constant ( or ) for all time except at where it is discontinuous. Since the derivative of any constant is zero, the derivative of the unit step has a non-zero value only at . The derivative of the unit step function is def... -- 1.4 The Delta Function -- The unit impulse or delta function, denoted as , is the derivative of the unit step . It is also defined as -- (1.33) -- and -- (1.34) -- To better understand the delta function , let us represent the unit step as shown in Fig ure 1.20 (a). -- Figure 1.20. Representation of the unit step as a limit -- The function of Figure 1.20 (a) becomes the unit step as . Figure 1.20 (b) is the derivative of Figure 1.20 (a), where we see that as , becomes unbounded, but the area of the rect angle remains . Therefore, in the limit, we can think of as a... -- Two useful properties of the delta function are the sampling property and the sifting property. -- 1.4.1 The Sampling Property of the Delta Function -- The sampling property of the delta function states that -- (1.35) -- or, when , -- (1.36) -- that is, multiplication of any function by the delta function results in sampling the func tion at the time instants where the delta function is not zero. The study of discrete-time systems is based on this property. -- Proof: -- Since then, -- (1.37) -- We rewrite as -- (1.38) -- Integrating (1.37) over the interval and using (1.38), we obtain -- (1.39) -- The first integral on the right side of (1.39) contains the constant term -- this can be written outside the integral, that is, -- (1.40) -- The second integral of the right side of (1.39) is always zero because -- and -- Therefore, (1.39) reduces to -- (1.41) -- Differentiating both sides of (1.41), and replacing with , we obtain -- (1.42) -- 1.4.2 The Sifting Property of the Delta Function. The sifting property of the delta function states that.

Sommario/riassunto

This text is primarily written for junior and senior undergraduates majoring in electrical and computer engineering. You will need this text if you are a student or working professional seeking to learn and/or review the basics of the Laplace and Z-transforms, the Fast Fourier Transform (FFT), state variables, and the design of analog and digital filters. Contains many real-world examples completely solved