

| | |
|-------------------------|--|
| 1. Record Nr. | UNINA9910770247003321 |
| Autore | Leitner Arielle |
| Titolo | An Invitation to Coarse Groups // by Arielle Leitner, Federico Vigolo |
| Pubbl/distr/stampa | Cham : , : Springer Nature Switzerland : , : Imprint : Springer, , 2023 |
| ISBN | 3-031-42760-2 |
| Edizione | [1st ed. 2023.] |
| Descrizione fisica | 1 online resource (249 pages) |
| Collana | Lecture Notes in Mathematics, , 1617-9692 ; ; 2339 |
| Altri autori (Persone) | VigoloFederico |
| Disciplina | 512.55 |
| Soggetti | Group theory Topological groups Lie groups Group Theory and Generalizations Topological Groups and Lie Groups Teoria de grups Grups topològics Grups de Lie Llibres electrònics |
| Lingua di pubblicazione | Inglese |
| Formato | Materiale a stampa |
| Livello bibliografico | Monografia |
| Nota di contenuto | Intro -- Preface: About this Book -- Contents -- List of Symbols -- 1 Introduction -- 1.1 Background and Motivation -- 1.2 List of Findings I: Basic Theory -- 1.3 List of Findings II: Selected Topics -- Part I Basic Theory -- 2 Introduction to the Coarse Category -- 2.1 Some Notation for Subsets and Relations -- 2.2 Coarse Structures -- 2.3 Coarse Structures via Partial Coverings -- 2.4 Controlled Maps -- 2.5 The Category of Coarse Spaces -- 2.6 Binary Products -- 2.7 Equi Controlled Maps -- 3 Properties of the Category of Coarse Spaces -- 3.1 Pull-Back and Push-Forward -- 3.2 Controlled Thickenings and Asymptoticity -- 3.3 Coarse Subspaces, Restrictions, Images and Quotients -- 3.4 Containments and Intersections of Coarse Subspaces -- 4 Coarse Groups -- 4.1 Preliminary: Group Objects in a Category -- 4.2 Coarse Groups: Definition, Notation and Examples -- 4.3 Equi Invariant Coarse Structures and Automatic Control -- 4.4 Making Sets into Coarse Groups -- 4.5 Coarse Groups are Determined Locally -- |

4.6 Summary: A Concrete Description of Coarse Groups -- 5 Coarse Homomorphisms, Subgroups and Quotients -- 5.1 Definitions and Examples of Coarse Homomorphisms -- 5.2 Properties of Coarse Homomorphisms -- 5.3 Coarse Subgroups -- 5.4 Coarse Quotients -- 6 Coarse Actions -- 6.1 Definition and Examples -- 6.2 Coarse Invariance and Equivariance -- 6.3 Coarse Action by Conjugation -- 6.4 Coarse Orbits -- 6.5 The Fundamental Observation of Geometric Group Theory -- 6.6 Quotient Coarse Actions -- 6.7 Coarse Quotient Actions of the Action by Left Multiplication -- 6.8 Coarse Cosets Spaces: Subsets -- 6.9 Coarse Cosets Spaces: Subgroups -- 7 Coarse Kernels -- 7.1 Coarse Preimages and Kernels -- 7.2 The Isomorphism Theorems -- 7.3 Short Exact Sequences of Coarse Groups -- 7.4 A Criterion for the Existence of Coarse Kernels -- 7.5 Some Comments and Questions.

Part II Selected Topics -- 8 Coarse Structures on Set-Groups -- 8.1 Connected Coarsifications of Set-Groups -- 8.2 Metric Coarsifications and Bi-Invariant Metrics -- 8.3 Some Examples of (Un)bounded Set-Groups -- 9 Coarse Structures on \mathbb{Z} -- 9.1 Coarse Structures Generated by Cayley Graphs -- 9.2 Coarse Structures Generated by Topologies -- 9.3 Some Questions -- 10 On Bi-Invariant Word Metrics -- 10.1 Computing the Cancellation Metric on Free Groups -- 10.2 The Canonical Coarsification of Finitely Generated Set-Groups and their Subgroups -- 11 A Quest for Coarse Groups that are not Coarsified Set-Groups -- 11.1 General Observations -- 11.2 A Conjecture on Coarse Groups that are not Coarsified Set-Groups -- 11.3 A Few More Questions -- 12 On Coarse Homomorphisms and Coarse Automorphisms -- 12.1 Elementary Constructions of Coarse Automorphisms -- 12.2 Coarse Homomorphisms into Banach Spaces -- 12.3 Hartnick-Schweitzer Quasimorphisms -- 12.4 Coarse Homomorphisms of Finitely Normally Generated Groups -- 13 Spaces of Controlled Maps -- 13.1 Fragmentary Coarse Structures -- 13.2 The Fragmentary Coarse Space of Controlled Maps -- 13.3 Coarse Actions Revisited -- 13.4 Frag-Coarse Groups of Controlled Transformations -- 13.5 Coarse Groups as Coarse Subgroups -- A Categorical Aspects of Coarse -- A.1 Limits and Colimits -- A.2 Subobjects and Quotients -- A.3 The Category of Coarse Groups -- A.4 The Category of Fragmentary Coarse Spaces -- A.5 Enriched Coarse Category -- A.6 The Pre-Coarse Category -- B Metric Groups and Quasifications -- B.1 Quasifications of Metric Spaces -- B.2 Groups in Metric Categories -- B.3 Quasi-Metric Groups -- C Extra Topics -- C.1 Computing Cancellation Metrics -- C.2 Proper Coarse Actions -- References -- Index.

Sommario/riassunto

This book lays the foundation for a theory of coarse groups: namely, sets with operations that satisfy the group axioms “up to uniformly bounded error”. These structures are the group objects in the category of coarse spaces, and arise naturally as approximate subgroups, or as coarse kernels. The first aim is to provide a standard entry-level introduction to coarse groups. Extra care has been taken to give a detailed, self-contained and accessible account of the theory. The second aim is to quickly bring the reader to the forefront of research. This is easily accomplished, as the subject is still young, and even basic questions remain unanswered. Reflecting its dual purpose, the book is divided into two parts. The first part covers the fundamentals of coarse groups and their actions. Here the theory of coarse homomorphisms, quotients and subgroups is developed, with proofs of coarse versions of the isomorphism theorems, and it is shown how coarse actions are related to fundamental aspects of geometric group theory. The second part, which is less self-contained, is an invitation to further research,

where each thread leads to open questions of varying depth and difficulty. Among other topics, it explores coarse group structures on set-groups, groups of coarse automorphisms and spaces of controlled maps. The main focus is on connections between the theory of coarse groups and classical subjects, including: number theory; the study of bi-invariant metrics on groups; quasimorphisms and stable commutator length; groups of outer automorphisms; and topological groups and their actions. The book will primarily be of interest to researchers and graduate students in geometric group theory, topology, category theory and functional analysis, but some parts will also be accessible to advanced undergraduates.
