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| Nota di contenuto       | Intro -- Preface -- Contents -- List of Symbols -- 1 Convergence of Sequences of Functions -- 1.1 Preliminaries and Notation -- 1.2 Pointwise and Uniform Convergence -- 1.3 Series of Functions -- 1.3.1 Power Series in the Complex Plane -- 1.3.2 Fourier Series -- 1.3.2.1 Dirichlet Kernel -- 1.3.2.2 Cesàro Means: Féjer Kernel -- 1.3.2.3 Poisson Kernel -- 1.3.3 Dirichlet Series -- 1.4 Exercises -- References -- 2 Locally Convex Spaces -- 2.1 Topological Preliminaries -- 2.1.1 Basic Definitions -- 2.1.2 Metric and Normed Spaces -- 2.2 Seminorms -- 2.2.1 Locally Convex Topology -- 2.2.2 Continuity -- 2.2.3 Metrizable Locally Convex Spaces -- 2.3 The Dual of a Locally Convex Space -- 2.4 Examples of Spaces -- 2.4.1 Space of Continuous Functions -- 2.4.2 Köthe Echelon Spaces -- 2.5 Normable Spaces -- 2.6 Two Theorems on Spaces of Continuous Functions -- 2.6.1 Stone-Weierstraß Theorem -- 2.6.2 Ascoli Theorem -- 2.7 A Short Introduction to Hilbert Spaces -- 2.8 Exercises -- References -- 3 Duality and Linear Operators -- 3.1 Hyperplanes -- 3.2 The Hahn-Banach Theorem -- 3.2.1 Analytic Version -- 3.2.2 Separation Theorems -- 3.2.3 Finite Dimensional Locally Convex Spaces -- 3.2.4 Banach Limits -- 3.3 Weak Topologies -- 3.4 The Bipolar Theorem -- 3.5 The Mackey-Arens Theorem -- 3.6 The Banach-Steinhaus Theorem -- 3.7 The Banach-Schauder Theorem -- 3.8 Topologies on the Space of Continuous Linear Mappings -- 3.9 Transpose of an Operator -- |

3.10 Exercises -- References -- 4 Spaces of Holomorphic and Differentiable Functions and Operators Between Them -- 4.1 Space of Holomorphic Functions -- 4.1.1 Locally Convex Structure -- 4.1.2 Representation as a Sequence Space -- 4.1.3 Montel Theorem -- 4.1.4 Dual of the Space of Entire Functions -- 4.2 Spaces of Differentiable Functions -- 4.3 Some Operators on Spaces of Functions -- 4.4 Exercises -- References.

5 Transitive and Mean Ergodic Operators -- 5.1 Transitive Operators -- 5.2 Mean Ergodic Operators -- 5.3 Examples -- 5.3.1 The Backward Shift -- 5.3.2 Composition Operators -- 5.3.3 Multiplication and Integration Operators -- 5.3.4 Differential Operators -- 5.4 Exercises -- References -- 6 Schwartz Distributions and Linear Partial Differential Operators -- 6.1 Test Functions and Distributions -- 6.1.1 Definition and Examples -- 6.1.2 Differentiation of Distributions -- 6.1.3 Multiplication of a Distribution by a C-Function -- 6.1.4 Support of a Distribution and Distributions with Compact Support -- 6.2 The Space of Rapidly Decreasing Functions -- 6.3 Fourier Transform on  $S(\mathbb{R}^N)$  -- 6.4 Tempered Distributions and the Fourier Transform -- 6.5 Linear Partial Differential Operators -- 6.5.1 Fundamental Solutions. The Malgrange-Ehrenpreis Theorem -- 6.5.2 Solutions of Linear PDEs -- 6.6 Exercises -- References -- References -- Index.

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## Sommario/riassunto

The aim of this work is to present, in a unified and reasonably self-contained way, certain aspects of functional analysis which are needed to treat function spaces whose topology is not derived from a single norm, their topological duals and operators between those spaces. We treat spaces of continuous, analytic and smooth functions as well as sequence spaces. Operators of differentiation, integration, composition, multiplication and partial differential operators between those spaces are studied. A brief introduction to Laurent Schwartz's theory of distributions and to Lars Hörmander's approach to linear partial differential operators is presented. The novelty of our approach lies mainly on two facts. First of all, we show all these topics together in an accessible way, stressing the connection between them. Second, we keep it always at a level that is accessible to beginners and young researchers. Moreover, parts of the book might be of interest for researchers in functional analysis and operator theory. Our aim is not to build and describe a whole, complete theory, but to serve as an introduction to some aspects that we believe are interesting. We wish to guide any reader that wishes to enter in some of these topics in their first steps. Our hope is that they learn interesting aspects of functional analysis and become interested to broaden their knowledge about function and sequence spaces and operators between them. The text is addressed to students at a master level, or even undergraduate at the last semesters, since only knowledge on real and complex analysis is assumed. We have intended to be as self-contained as possible, and wherever an external citation is needed, we try to be as precise as we can. Our aim is to be an introduction to topics in, or connected with, different aspects of functional analysis. Many of them are in some sense classical, but we tried to show a unified direct approach; some others are new. This is why parts of these lectures might be of some interest even for researchers in related areas of functional analysis or operator theory. There is a full chapter about transitive and mean ergodic operators on locally convex spaces. This material is new in book form. It is a novel approach and can be of interest for researchers in the area.

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