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Autore	Golasiski Marek
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Sommario/riassunto

This monograph deals with matrix manifolds, i.e., manifolds for which there is a natural representation of their elements as matrix arrays. Classical matrix manifolds (Stiefel, Grassmann and flag manifolds) are studied in a more general setting. It provides tools to investigate matrix varieties over Pythagorean formally real fields. The presentation of the book is reasonably self-contained. It contains a number of nontrivial results on matrix manifolds useful for people working not only in differential geometry and Riemannian geometry but in other areas of mathematics as well. It is also designed to be readable by a graduate student who has taken introductory courses in algebraic and differential geometry.