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Sommario/riassunto

This monograph develops a theory of continuous and differentiable functions, called monogenic functions, in the sense of Gateaux functions taking values in some vector spaces with commutative multiplication. The study of these monogenic functions in various commutative algebras leads to a discovery of new ways of solving boundary value problems in mathematical physics. The book consists of six parts: Part I presents some preliminary notions and introduces various concepts of differentiable mappings of vector spaces. Part II - V is devoted to the study of monogenic functions in various spaces with commutative multiplication, namely, three dimensional commutative algebras with two-dimensional radical, finite-dimensional commutative associative algebras, infinite-dimensional vector spaces associated with the three-dimensional Laplace equation and infinite-dimensional vector spaces associated with axial-symmetric potential fields. Part VI presents some boundary value problems for axial-symmetric potential fields and develops effective analytic methods of solving these boundary value problems with various applications in mathematical physics. Graduate students and researchers alike benefit from this book.