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Sommario/riassunto

This volume collects the lecture notes of the school TIME2019 (Treasures in Mathematical Encounters). The aim of this book is manifold, it intends to overview the wide topic of algebraic curves and surfaces (also with a view to higher dimensional varieties) from different aspects: the historical development that led to the theory of algebraic surfaces and the classification theorem of algebraic surfaces by Castelnuovo and Enriques; the use of such a classical geometric approach, as the one introduced by Castelnuovo, to study linear systems of hypersurfaces; and the algebraic methods used to find implicit equations of parametrized algebraic curves and surfaces, ranging from classical elimination theory to more modern tools involving syzygy theory and Castelnuovo-Mumford regularity. Since our subject has a long and venerable history, this book cannot cover all the details of this broad topic, theory and applications, but it is meant to serve as a guide for both young mathematicians to approach the subject from a classical and yet computational perspective, and for experienced researchers as a valuable source for recent applications.
