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Sommario/riassunto

This book is about extending index theory to some examples where non-Fredholm operators arise. It focuses on one aspect of the problem of what replaces the notion of spectral flow and the Fredholm index when the operators in question have zero in their essential spectrum. Most work in this topic stems from the so-called Witten index that is discussed at length here. The new direction described in these notes is the introduction of `spectral flow beyond the Fredholm case'. Creating a coherent picture of numerous investigations and scattered notions of the past 50 years, this work carefully introduces spectral flow, the Witten index and the spectral shift function and describes their relationship. After the introduction, Chapter 2 carefully reviews Double Operator Integrals, Chapter 3 describes the class of so-called p-relative trace class perturbations, followed by the construction of Krein's spectral shift function in Chapter 4. Chapter 5 reviews the analytic approach tospectral flow, culminating in Chapter 6 in the main abstract result of the book, namely the so-called principal trace formula. Chapter 7 completes the work with illustrations of the main results using explicit computations on two examples: the Dirac operator in Rd, and a differential operator on an interval. Throughout, attention is paid to the history of the subject and earlier references are provided accordingly. The book is aimed at experts in index theory as well as newcomers to the field.