

1. Record Nr.	UNINA9910631079903321
Autore	Schenck Hal
Titolo	Algebraic foundations for applied topology and data analysis / / Hal Schenck
Pubbl/distr/stampa	Cham, Switzerland : , : Springer, , [2022] ©2022
ISBN	9783031066641 9783031066634
Descrizione fisica	1 online resource (231 pages)
Collana	Mathematics of Data ; ; v.1
Disciplina	514.2
Soggetti	Algebraic topology Topological algebras Topologia algebraica Àlgebres topològiques Llibres electrònics
Lingua di pubblicazione	Inglese
Formato	Materiale a stampa
Livello bibliografico	Monografia
Nota di bibliografia	Includes bibliographical references and index.
Nota di contenuto	Intro -- Preface -- Contents -- 1 Linear Algebra Tools for Data Analysis -- 1.1 Linear Equations, Gaussian Elimination, Matrix Algebra -- 1.2 Vector Spaces, Linear Transformations, Basis and Change of Basis -- 1.2.1 Basis of a Vector Space -- 1.2.2 Linear Transformations -- 1.2.3 Change of Basis -- 1.3 Diagonalization, Webpage Ranking, Data and Covariance -- 1.3.1 Eigenvalues and Eigenvectors -- 1.3.2 Diagonalization -- 1.3.3 Ranking Using Diagonalization -- 1.3.4 Data Application: Diagonalization of the Covariance Matrix -- 1.4 Orthogonality, Least Squares Fitting, Singular Value Decomposition -- 1.4.1 Least Squares -- 1.4.2 Subspaces and Orthogonality -- 1.4.3 Singular Value Decomposition -- 2 Basics of Algebra: Groups, Rings, Modules -- 2.1 Groups, Rings and Homomorphisms -- 2.1.1 Groups -- 2.1.2 Rings -- 2.2 Modules and Operations on Modules -- 2.2.1 Ideals -- 2.2.2 Tensor Product -- 2.2.3 Hom -- 2.3 Localization of Rings and Modules -- 2.4 Noetherian Rings, Hilbert Basis Theorem, Varieties -- 2.4.1 Noetherian Rings -- 2.4.2 Solutions to a Polynomial System: Varieties -- 3 Basics of Topology: Spaces and Sheaves -- 3.1

Topological Spaces -- 3.1.1 Set Theory and Equivalence Relations --
3.1.2 Definition of a Topology -- 3.1.3 Discrete, Product, and Quotient
Topologies -- 3.2 Vector Bundles -- 3.3 Sheaf Theory -- 3.3.1
Presheaves and Sheaves -- 3.3.2 Posets, Direct Limit, and Stalks --
3.3.3 Morphisms of Sheaves and Exactness -- 3.4 From Graphs to
Social Media to Sheaves -- 3.4.1 Spectral Graph Theory -- 3.4.2 Heat
Diffusing on a Wire Graph -- 3.4.3 From Graphs to Cellular Sheaves --
4 Homology I: Simplicial Complexes to Sensor Networks -- 4.1
Simplicial Complexes, Nerve of a Cover -- 4.1.1 The Nerve of a Cover
-- 4.2 Simplicial and Singular Homology -- 4.2.1 Singular homology --
4.3 Snake Lemma and Long Exact Sequence in Homology.
4.3.1 Maps of complexes, Snake Lemma -- 4.3.2 Chain Homotopy --
4.4 Mayer-Vietoris, Rips and ech Complex, Sensor Networks -- 4.4.1
Mayer-Vietoris Sequence -- 4.4.2 Relative Homology -- 4.4.3 ech
Complex and Rips Complex -- 5 Homology II: Cohomology to Ranking
Problems -- 5.1 Cohomology: Simplicial, ech, de Rham Theories --
5.1.1 Simplicial Cohomology -- 5.1.2 ech Cohomology -- 5.1.3 de
Rham Cohomology -- 5.2 Ranking, the Netflix Problem, and Hodge
Theory -- 5.2.1 Hodge Decomposition -- 5.2.2 Application to Ranking
-- 5.3 CW Complexes and Cellular Homology -- 5.4 Poincaré and
Alexander Duality: Sensor Networks Revisited -- 5.4.1 Statement of
Theorems and Examples -- 5.4.2 Alexander Duality: Proof -- 5.4.3
Sensor Networks Revisited -- 5.4.4 Poincaré Duality -- 6 Persistent
Algebra: Modules Over a PID -- 6.1 Principal Ideal Domains and
Euclidean Domains -- 6.2 Rational Canonical Form of a Matrix -- 6.3
Linear Transformations, $K[t]$ -Modules, Jordan Form -- 6.4 Structure of
Abelian Groups and Persistent Homology -- 6.4.1 Z-Graded Rings -- 7
Persistent Homology -- 7.1 Barcodes, Persistence Diagrams, Bottleneck
Distance -- 7.1.1 History -- 7.1.2 Persistent Homology and the
Barcode -- 7.1.3 Computation of Persistent Homology -- 7.1.4 Alpha
and Witness Complexes -- 7.1.5 Persistence Diagrams -- 7.1.6 Metrics
on Diagrams -- 7.2 Morse Theory -- 7.3 The Stability Theorem -- 7.4
Interleaving and Categories -- 7.4.1 Categories and Functors -- 7.4.2
Interleaving -- 7.4.3 Interleaving Vignette: Merge Trees -- 7.4.4 Zigzag
Persistence and Quivers -- 8 Multiparameter Persistent Homology --
8.1 Definition and Examples -- 8.1.1 Multiparameter Persistence -- 8.2
Graded Algebra, Hilbert Function, Series, Polynomial -- 8.2.1 The
Hilbert Function -- 8.2.2 The Hilbert Series -- 8.3 Associated Primes
and Zn-Graded Modules -- 8.3.1 Geometry of Sheaves.
8.3.2 Associated Primes and Primary Decomposition -- 8.3.3 Additional
Structure in the Zn-Graded Setting -- 8.4 Filtrations and Ext -- 9
Derived Functors and Spectral Sequences -- 9.1 Injective and Projective
Objects, Resolutions -- 9.1.1 Projective and Injective Objects -- 9.1.2
Resolutions -- 9.2 Derived Functors -- 9.2.1 Categories and Functors
-- 9.2.2 Constructing Derived Functors -- 9.2.3 Ext -- 9.2.4 The
Global Sections Functor -- 9.2.5 Acyclic Objects -- 9.3 Spectral
Sequences -- 9.3.1 Total Complex of Double Complex -- 9.3.2 The
Vertical Filtration -- 9.3.3 Main Theorem -- 9.4 Pas de Deux: Spectral
Sequences and Derived Functors -- 9.4.1 Resolution of a Complex --
9.4.2 Grothendieck Spectral Sequence -- 9.4.3 Comparing Cohomology
Theories -- 9.4.4 Cartan-Eilenberg Resolution -- A Examples of
Software Packages -- A.1 Covariance and Spread of Data via R -- A.2
Persistent Homology via scikit-tda -- A.3 Computational Algebra via
Macaulay2 -- A.4 Multiparameter Persistence via RIVET -- Bibliography
-- Index.
