

1. Record Nr.	UNINA9910595027203321
Autore	Arabia Alberto
Titolo	Introduction to isospectrality / / Alberto Arabia
Pubbl/distr/stampa	Cham, Switzerland : , : Springer, , [2022] ©2022
ISBN	9783031171239 9783031171222
Descrizione fisica	1 online resource (247 pages)
Collana	Universitext
Disciplina	605
Soggetti	Spectral geometry Geometria espectral Llibres electrònics
Lingua di pubblicazione	Inglese
Formato	Materiale a stampa
Livello bibliografico	Monografia
Nota di bibliografia	Includes bibliographical references and index.
Nota di contenuto	Intro -- Preface -- The Plan of the Book -- Acknowledgements -- Contents -- Chapter 1 Introduction -- 1 Spectral Geometry -- 1.1 Visible Light Spectroscopy -- 1.2 From Light to Sound -- 1.2.1 The Fourier Decomposition of Periodic Functions -- 1.2.2 Comment. -- 1.2.3 Fourier Transform of Functions -- 1.2.4 Comment. -- 1.3 The Wave Equation -- 1.3.1 Strings -- 1.3.2 Membranes and Manifolds -- 1.3.3 Manifolds with Boundary -- 1.3.4 On the Constant ' -- 1.4 Natural Vibrations and Natural Sounds -- 1.4.1 Natural Vibrations. -- 1.4.2 Proposition. -- 1.4.3 Natural Sounds. -- 1.5 The Spectrum of a Riemannian Manifold -- 1.5.1 Notation and Terminology -- 1.5.2 Modes of Vibration -- 1.5.3 The Spectral Theorem -- 1.5.4 Theorem (Spectral Theorem) -- 1.6 Solutions of the Wave Equation and Natural Sounds -- 1.6.1 Solutions of theWave Equation. -- 1.6.2 Natural Sounds. -- 1.6.3 Definition. -- 1.6.4 Proposition. -- 1.6.5 Comment. -- 1.7 Comparing Natural Sounds -- 1.7.1 Definition. -- 1.7.2 Theorem. -- 1.7.3 Comments -- 1.7.4 Conclusion. -- 2 Isospectrality and Isometry -- 2.1 Direct and Inverse Problems in Spectral Geometry -- 2.2 Interpreting Kac's Question -- 2.3 Boundary Conditions and Irreducibility -- 3 The Spectra of Strings -- 3.1 Strings -- 3.2 Smooth Functions on Strings -- 3.3 The Extended Spectra of Strings -- 3.3.1

Proposition. -- 3.3.2 Exercises. -- 4 The Spectra of Rectangular Membranes -- 4.1 Domains and Membranes -- 4.2 Smooth Functions on Membranes -- 4.3 Rectangular Membranes -- 4.4 Dirichlet and Neumann Spectra of Rectangular Membranes -- 4.5 Notable Properties of Spec (D) -- 4.5.1 Proposition. -- 5 Some General Results of Spectral Geometry -- 5.1 Weyl's Law. -- 5.1.1 Exercise. -- 5.2 Local/Global Uniqueness of Eigenfunction -- 5.2.1 Theorem (Bers). -- 5.2.2 Corollary (Cheng). -- 5.3 Nodal Sets. -- 5.3.1 Nodal Domains of an Eigenfunction. -- 5.3.2 Theorem (R. Courant). -- 5.3.3 Corollary. -- 5.3.4 Example of Eigenfunctions for the First Two Eigenvalues -- 6 Construction of Isospectral Flat Surfaces -- 6.1 Sunada's Method -- 6.1.1 The Trace Formula -- 6.1.2 Proposition (The Trace Formula). -- 6.1.3 Gassmann Triples and Sunada's Theorem -- 6.1.4 Definition. -- 6.1.5 Theorem (Sunada [70], 1985) -- 6.1.6 Comments. -- 6.2 Transplantations and Enhancements of Sunada's Method -- 6.2.1 Theorem. -- 6.2.2 Comments. -- 6.2.3 Buser and Bérard Enhancements of Sunada's Method -- 6.3 Construction of Isospectral Surfaces -- 6.3.1 Examples of Gassmann Triples -- 6.4 Cayley Graphs and Tessellated Buser Surfaces -- 6.4.1 Schreier Graphs. -- 6.5 Buser Flat Surfaces -- 6.6 The Gordon-Webb-Wolpert Domains -- 6.7 Transplantations -- 6.8 Bérard-Buser's Surfaces and Beyond -- 7 General References -- Chapter 2 The Wave Equation on Flat Manifolds -- 1 Flat Riemannian Manifolds -- 1.1 Euclidean Atlases and Flat Manifolds -- 1.1.1 Terminology. -- 1.1.2 Definitions. -- 1.1.3 Exercise. -- 1.1.4 Theorem. -- 1.1.5 Definition. -- 1.2 The Path-Distance on a Flat Manifold -- 1.3 Functions on a Flat Manifold -- 1.3.1 Definition. -- 1.3.2 Proposition. -- 2 The Wave Equation on a Flat Manifold -- 2.1 The Laplacian on a Flat Manifold -- 2.1.1 Theorem. -- 2.2 The Wave Equation and the Spectrum of a Flat Manifold -- 2.2.1 Natural Sounds of a Flat Manifold -- 2.2.2 Comment. -- 2.2.3 Definition. -- 2.2.4 Theorem. -- 3 Flat Surfaces with Piecewise Linear Boundary -- 3.1 Open Euclidean Sets with PL-Boundary -- 3.1.1 PLB-Domains of $\mathbb{R}^2$. -- 3.1.2 Differentiability on PLB-Domains of $\mathbb{R}^2$. -- 3.1.3 Proposition. -- 3.1.4 The Normal Derivative on the Boundary -- 3.2 Differentiability on PLB-Domains of $\mathbb{R}^2$. -- 3.2.1 Definition. -- 3.2.2 Theorem and Definition. -- 3.2.3 Open PLB-domains. -- 3.3 PLB-Surfaces -- 3.3.1 Definition. -- 3.3.2 The Interior Gluing Data -- 3.4 PLB-Surface Defined by Gluing Open PLB-Domains -- 3.4.1 Definition. -- 3.4.2 Theorem. -- 3.4.3 Complete PLB-Atlas of -- 3.4.4 Topological Separation of $M()$ -- 3.4.5 Proposition. -- 3.4.6 Example. -- 3.4.7 Theorem and Definitions. -- 3.4.8 Comment. -- 3.4.9 Examples of Constructions of PLB-Surfaces -- 3.5 Differentiable Functions and the Laplacian on a PLB-Surface -- 3.5.1 Definitions. -- 3.5.2 Proposition and Definitions -- 3.5.3 Theorem. -- 4 Group Quotients of PLB-Surfaces -- 4.1 Theorem. -- 5 The Wave Equation of a PLB-Surface -- 5.1 The Extended Spectrum of a PLB-Surface -- 5.2 Isospectrality of PLB-Surfaces -- 5.2.1 Definition. -- 5.2.2 Theorem -- 6 Orbifold and Folding Boundaries -- 6.1 On the Action of a General Isometry of a PLB-Surface -- 6.1.1 Proposition. -- 6.1.2 Comment. -- 6.1.3 Corollary. -- 6.1.4 Comment. -- Chapter 3 The Sunada-Bérard-Buser Method -- 1 The Sunada-Bérard-Buser Theorem for PLB-Surfaces -- 1.1 A Warning on the Notation for Left and Right Actions -- 1.2 The Sunada-Bérard-Buser Method in Brief -- 1.2.1 Transplantation Theorem -- 1.2.2 The Sunada-Bérard-Buser Theorem. -- 2 PLB-Surfaces with Specified Group of Isometries -- 2.1 The Cayley Graph of a Group -- 2.1.1 Convention. -- 2.1.2 Definition. -- 2.1.3 Examples of Cayley Graphs of Groups -- 2.1.4 Lemma. -- 2.1.5 Definition -- 2.1.6 Proposition. -- 2.1.7 Exercise. -- 2.2 The

Schreier Graph of a Right G-Set -- 2.2.1 The Category of Right G-Sets
 -- 2.2.2 Definitions. -- 2.2.3 Proposition. -- 2.2.4 The Schreier Graph
 Functor on the Category Set-G -- 2.2.5 Definitions. -- 2.2.6 Theorem.
 -- 2.2.7 Examples of Schreier Graphs of Irreducible Right G-Sets --
 The Schreier graph $(R \setminus D4, S)$ -- 2.2.8 Comments. -- 2.3 Gassmann
 Triples -- 2.3.1 Proposition. -- 2.3.2 Exercises. -- 2.3.3 The Perlis-
 Brooks-Tse Gassmann Triple $(G, 1, 2)$.
 2.3.4 Proposition. -- 2.3.5 Comment. -- 2.3.6 The Schreier Graphs
 Associated with $(G, 1, 2)$ -- 2.4 Piecewise Strings Associated with
 Schreier Graphs -- 2.4.1 One-Dimensional Tiles -- 2.4.2 The Buser
 Space $M(1, X, S)$ -- 2.4.3 Remark. -- 2.4.4 Definition. -- 2.3.3
 Definition. -- 2.4.5 Theorem. -- 2.5 PLB-Surfaces Associated with
 Schreier Graphs -- 2.5.1 Two-Dimensional Tiles -- 2.5.2 The Buser
 Surfaces -- 2.5.3 Comments. -- 2.5.4 Examples -- 2.5.5 Theorem. --
 2.6 Buser Manifolds Associated With Schreier Graphs -- 2.7 Volume and
 Perimeter of -- 2.7.1 Proposition. -- 2.8 On the Buser Method -- 2.8.1
 Irregular Tiles. -- 2.8.2 Proposition. -- 2.8.3 Comment. -- 3 Non-
 Isometric Isospectral Buser Spaces -- 3.1 Theorem. -- 3.1.1 Comment.
 -- 3.2 Buser Isospectral Spaces -- 4 Transplantations in Buser Surfaces
 -- 4.1 Transplantations and 2×1 -Orbits -- 4.1.1 Finding
 Transplantations for General Triples $(G, 1, 2)$ -- 4.2 Transplantation
 Matrices on Buser's Isospectral Spaces -- 4.2.1 Proposition. -- 4.3 The
 Name of the Game -- 4.3.1 Manual Check of the Differentiability of
 Transplantations -- 4.3.2 Comment. -- 4.3.3 Exercise. -- 4.3.4
 Exercise. -- 4.4 Reflections in Buser Surfaces -- 4.4.1 Definition. --
 4.4.2 Proposition. -- 4.4.3 Comment. -- Chapter 4 The Gordon-Webb-
 Wolpert Isospectral Domains -- 1 A Special Reflection on Buser
 Surfaces -- 1.1 Notation. -- 1.2 The Reflections i . -- 1.3 The GWW
 Domains D_i -- 1.3.1 Folding Edges. -- 1.3.2 Comment. -- 1.3.3
 Proposition (GWW[39]) -- 2 Transplantations of Continuous Functions
 -- 2.1 Proposition. -- 2.2 Double-Sided Tessellation of Buser Surfaces
 -- 2.3 The Transplantation Matrix for Continuous Functions on
 GWW Domains -- 3 Lifting Smooth Functions of GWW Domains -- 3.1
 The Euclidean Half-Space -- 3.1.1 Theorem. -- 3.1.2 Comment. -- 3.2
 The Spectrum of $D := \bigcup M$, for $M := \mathbb{R}^2$ and $2(x, y) := (x, y)$ -- 3.2.1
 Corollary.
 3.3 Different Natures of the Boundaries on GWW Domains -- 3.4 Mixed
 Boundary Conditions on GWW Domains -- 3.4.1 Definitions. -- 3.4.2
 Theorem -- 4 Isospectrality of Gordon-Webb-Wolpert Domains -- 4.1
 The Mixed Extended Spectrum of a GWW Domain -- 4.1.1 Theorem
 (GWW[39]). -- 5 Transplantations in Gordon-Webb-Wolpert Domains --
 5.1 General Transplantation Setup -- 5.2 Transplantation Matrix T_N --
 5.3 Transplantation Matrix T_D -- 5.4 Transplantation Recipe -- 5.5
 Changing the Tile's Shape -- 5.5.1 The Hen and the Arrow. -- 5.5.2
 Exercises. -- 6 Research in Isospectrality -- 6.1 Important Dates in
 Isospectrality. -- 6.2 Other Routes of Research -- 6.2.1 On
 Transplantations -- 6.2.2 On the Arithmetical Approach to
 Isospectrality -- 6.2.3 On Isospectral Simply-Connected Closed
 Manifolds -- 6.2.4 On Isospectral Compact Flat Manifolds -- 6.3 On
 Non-Strongly Isospectral Manifolds -- 6.3.1 Miscellanea -- Appendix A
 Linear Representations of Finite Groups and Almost-Conjugate
 Subgroups -- 1 The Category of Linear Representations of a Group --
 1.1 The Group Ring. -- 1.1.1 Proposition. -- 1.1.2 Definition. -- 1.2
 Category of M -Modules -- 1.2.1 Definitions. -- 1.2.2 Exchange
 Principle -- 1.3 Permutation Representations -- 1.3.1 Vector Space
 Spanned by a Set -- 1.3.2 M -Module Spanned by a M -Set -- 1.3.3
 Examples -- 1.4 Symmetrization Operator -- 1.4.1 Definition. -- 1.4.2
 Proposition -- 1.5 Operations on M -Modules -- 1.5.1 Products and

Direct Sums -- 1.5.2 Submodules and Quotient Modules -- 1.5.3
Proposition. -- 1.5.4 The M-Module $\text{Hom}_Q(_, _)$ -- 1.5.5 Proposition. --
1.5.6 The Module $(_) := \text{Hom}_Q(_, Q)$ -- 1.5.7 The Functor $\text{Hom}_M(Q[G], _)$ -- 1.5.8 Proposition. -- 1.6 Semisimplicity of Finite-Dimensional
Representations -- 1.6.1 Definitions -- 1.6.2 Theorem. -- 1.6.3
Exercise. -- 1.7 Unitary M-Modules -- 1.7.1 Definitions. -- 1.7.2
Proposition. -- 1.7.3 Unitary Equivalences.
1.7.4 Proposition.
