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Nota di contenuto	Intro -- Preface -- For Whom Is This Book Written? -- Who Can Understand This Book? -- What Is at Stake? -- Who Has Contributed? -- Preface to the Translation -- Contents -- Introduction: The Four Big Topics of This Book -- The Configuration of Mathematics-or: Designing Mathematical Theories -- To Define Is Hard Work! -- Is a Mathematical Proof Beyond Reproach? -- From Confusion to Clarity -- Growing Insight in the Formative Power of Definitions in Mathematics -- The Change, Seen from a Philosophical Viewpoint -- The Formation of Mathematics-or: The Transformations of Analysis -- The Foundational Years -- An Era of Pomposity: Algebraic Analysis -- The Implosion of Algebraic Analysis-and a First Attempt to Replace It -- Implementation of a Capricious Value Analysis -- Outlook: Axiomatics, Analysis Within Set-Theory and a New Kind of Formal Calculation -- The First Mathematical News in This Book: The Archetype of Today's Analysis (from Cauchy) -- The Second Mathematical News in This Book: A Third Construction of the Real Numbers (by Weierstraß) -- The Historiographical Hallmarks of This Book -- In Substance -- In Method

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