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Autore	Nicola Fabio
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Sommario/riassunto

The purpose of this monograph is to offer an accessible and essentially self-contained presentation of some mathematical aspects of the Feynman path integral in non-relativistic quantum mechanics. In spite of the primary role in the advancement of modern theoretical physics and the wide range of applications, path integrals are still a source of challenging problem for mathematicians. From this viewpoint, path integrals can be roughly described in terms of approximation formulas for an operator (usually the propagator of a Schrödinger-type evolution equation) involving a suitably designed sequence of operators. In keeping with the spirit of harmonic analysis, the guiding theme of the

book is to illustrate how the powerful techniques of time-frequency analysis - based on the decomposition of functions and operators in terms of the so-called Gabor wave packets – can be successfully applied to mathematical path integrals, leading to remarkable results and paving the wayto a fruitful interaction. This monograph intends to build a bridge between the communities of people working in time-frequency analysis and mathematical/theoretical physics, and to provide an exposition of the present novel approach along with its basic toolkit. Having in mind a researcher or a Ph.D. student as reader, we collected in Part I the necessary background, in the most suitable form for our purposes, following a smooth pedagogical pattern. Then Part II covers the analysis of path integrals, reflecting the topics addressed in the research activity of the authors in the last years.