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Autore	Mochizuki Takuro <1972->
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Sommario/riassunto

This book studies a class of monopoles defined by certain mild
 conditions, called periodic monopoles of generalized Cherkis–Kapustin
 (GCK) type. It presents a classification of the latter in terms of
 difference modules with parabolic structure, revealing a kind of
 Kobayashi–Hitchin correspondence between differential geometric
 objects and algebraic objects. It also clarifies the asymptotic behaviour
 of these monopoles around infinity. The theory of periodic monopoles
 of GCK type has applications to Yang–Mills theory in differential
 geometry and to the study of difference modules in dynamical algebraic
 geometry. A complete account of the theory is given, including major
 generalizations of results due to Charbonneau, Cherkis, Hurtubise,
 Kapustin, and others, and a new and original generalization of the
 nonabelian Hodge correspondence first studied by Corlette, Donaldson,
 Hitchin and Simpson. This work will be of interest to graduate students
 and researchers in differential and algebraic geometry, as well as in
 mathematical physics.
