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Sommario/riassunto

Algebraic Topology is an introductory textbook based on a class for advanced high-school students at the Stanford University Mathematics Camp (SUMaC) that the authors have taught for many years. Each chapter, or lecture, corresponds to one day of class at SUMaC. The book begins with the preliminaries needed for the formal definition of a surface. Other topics covered in the book include the classification of surfaces, group theory, the fundamental group, and homology. This book assumes no background in abstract algebra or real analysis, and the material from those subjects is presented as needed in the text. This makes the book readable to undergraduates or high-school students who do not have the background typically assumed in an algebraic topology book or class. The book contains many examples and exercises, allowing it to be used for both self-study and for an

introductory undergraduate topology course.
