1. Record Nr. UNINA9910480363703321 Autore Zeidler Eberhard Titolo Applied Functional Analysis [[electronic resource]]: Applications to Mathematical Physics / / by Eberhard Zeidler New York, NY:,: Springer New York:,: Imprint: Springer,, 1995 Pubbl/distr/stampa **ISBN** 1-4612-0815-7 Edizione [1st ed. 1995.] Descrizione fisica 1 online resource (XXIX, 481 p.) Collana Applied Mathematical Sciences, , 0066-5452; ; 108 Classificazione 46Bxx Disciplina 515 Soggetti Mathematical analysis Analysis (Mathematics) System theory Calculus of variations **Analysis** Systems Theory, Control Calculus of Variations and Optimal Control: Optimization Lingua di pubblicazione Inglese Formato Materiale a stampa Livello bibliografico Monografia Note generali Bibliographic Level Mode of Issuance: Monograph Nota di bibliografia Includes bibliographical references and index. Nota di contenuto 1 Banach Spaces and Fixed-Point Theorems -- 1.1 Linear Spaces and Dimension -- 1.2 Normed Spaces and Convergence -- 1.3 Banach Spaces and the Cauchy Convergence Criterion -- 1.4 Open and Closed Sets -- 1.5 Operators -- 1.6 The Banach Fixed-Point Theorem and the Iteration Method -- 1.7 Applications to Integral Equations -- 1.8 Applications to Ordinary Differential Equations -- 1.9 Continuity --1.10 Convexity -- 1.11 Compactness -- 1.12 Finite-Dimensional Banach Spaces and Equivalent Norms -- 1.13 The Minkowski Functional and Homeomorphisms -- 1.14 The Brouwer Fixed-Point Theorem --1.15 The Schauder Fixed-Point Theorem -- 1.16 Applications to Integral Equations -- 1.17 Applications to Ordinary Differential Equations -- 1.18 The Leray-Schauder Principle and a priori Estimates -- 1.19 Sub- and Supersolutions, and the Iteration Method in Ordered Banach Spaces -- 1.20 Linear Operators -- 1.21 The Dual Space --

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Sommario/riassunto

A theory is the more impressive, the simpler are its premises, the more distinct are the things it connects, and the broader is its range of applicability. Albert Einstein There are two different ways of teaching mathematics, namely, (i) the systematic way, and (ii) the application-oriented way. More precisely, by (i), I mean a systematic presentation of the material governed by the desire for mathematical perfection and completeness of the results. In contrast to (i), approach (ii) starts out from the question "What are the most important applications?" and then tries to answer this question as quickly as possible. Here, one walks directly on the main road and does not wander into all the nice and interesting side roads. The present book is based on the second approach. It is addressed to undergraduate and beginning graduate

students of mathematics, physics, and engineering who want to learn how functional analysis elegantly solves mathematical problems that are related to our real world and that have played an important role in the history of mathematics. The reader should sense that the theory is being developed, not simply for its own sake, but for the effective solution of concrete problems. viii Preface This introduction to functional analysis is divided into the following two parts: Part I: Applications to mathematical physics (the present AMS Vol. 108); Part II: Main principles and their applications (AMS Vol. 109).