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## Sommario/riassunto

The derivative and the integral are the fundamental notions of calculus. Though there is essentially only one derivative, there are a variety of integrals, developed over the years for a variety of purposes, and this book describes them. No other single source treats all of the integrals of Cauchy, Riemann, Riemann-Stieltjes, Lebesgue, Lebesgue-Stieltjes, Henstock-Kurzweil, Weiner, and Feynman. The basic properties of each are proved, their similarities and differences are pointed out, and the reason for their existence and their uses are given. Historical information is plentiful. Advanced undergraduate mathematics majors, graduate students, and faculty members are the audience for the book. Even experienced faculty members are unlikely to be aware of all of the integrals in the Garden of Integrals and the book provides an opportunity to see them and appreciate the richness of the idea of integral. Professor Burke's clear and well-motivated exposition makes this book a joy to read.

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