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## Sommario/riassunto

This book provides a thorough introduction to one of the most efficient approximation methods for the analysis and solution of problems in theoretical physics and applied mathematics. It is written with practical needs in mind and contains a discussion of 50 problems with solutions, of varying degrees of difficulty. The problems are taken from quantum mechanics, but the method has important applications in any field of science involving second order ordinary differential equations. The power of the asymptotic solution of second order differential equations is demonstrated, and in each case the authors clearly indicate which concepts and results of the general theory are needed to solve a

particular problem. This book will be ideal as a manual for users of the phase-integral method, as well as a valuable reference text for experienced research workers and graduate students.

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