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Sommario/riassunto

In this thesis, we study the regularity of optimal transport maps and its applications to the semi-geostrophic system. The first two chapters survey the known theory, in particular there is a self-contained proof of Brenier' theorem on existence of optimal transport maps and of Caffarelli's Theorem on Holder continuity of optimal maps. In the third and fourth chapter we start investigating Sobolev regularity of optimal transport maps, while in Chapter 5 we show how the above mentioned results allows to prove the existence of Eulerian solution to the semi-geostrophic equation. In Chapter 6 we prove partial regularity of optimal maps with respect to a generic cost functions (it is well known that in this case global regularity can not be expected). More precisely we show that if the target and source measure have smooth densities the optimal map is always smooth outside a closed set of measure zero.
