1. Record Nr. UNINA9910254062303321

Autore	Eynard Bertrand
Titolo	Counting surfaces : CRM Aisenstadt Chair lectures / / by Bertrand Eynard
Pubbl/distr/stampa	Basel : , : Springer Basel : , : Imprint : Birkhäuser, , 2016
ISBN	3-7643-8797-1
Edizione	[1st ed. 2016.]
Descrizione fisica	1 online resource (427 p.)
Collana	Progress in Mathematical Physics, , 1544-9998 ; ; 70
Disciplina	515.93
Soggetti	Algebraic geometry Combinatorics Algebraic Geometry
Lingua di pubblicazione	Inglese
Formato	Materiale a stampa
Livello bibliografico	Monografia
Note generali	Description based upon print version of record.
Nota di bibliografia	Includes bibliographical references and index.
Nota di contenuto	Preface; The CRM and Aisenstadt Chair; Acknowledgments; Topic of the Book; What Is Not Done in This Book; Contents; 1 Maps and Discrete Surfaces; 1.1 Gluing Polygons; 1.1.1 Intuitive Definition; 1.1.2 Formal Definition; 1.1.2.1 Definition with Permutations; 1.1.2.2 Definition with Embedded Graphs on Surfaces; 1.1.3 Topology; 1.1.4 Symmetry Factor; 1.2 Generating Functions for Counting Maps; 1.2.1 Maps with Fixed Number of Vertices; 1.2.2 Fixed Boundary Lengths; 1.2.3 Redundancy of the Parameters; 1.2.4 All Genus; 1.2.5 Non Connected Maps; 1.2.6 Rooted Maps: One Boundary; 1.3 Tutte's Equations 1.3.1 Planar Case: The Disk1.3.2 Higher Genus Tutte Equations; 1.4 Exercises; 2 Formal Matrix Integrals; 2.1 Definition of a Formal Matrix Integral; 2.1.1 Introductory Example: 1-Matrix Model and Quartic Potential; 2.1.2 Comparison with Convergent Integrals; 2.1.3 Formal Integrals, General Case; 2.2 Wick's Theorem and Combinatorics; 2.2.1 Generalities About Wick's Theorem; 2.2.1.1 Graphs; 2.2.1.2 Symmetry Factors; 2.2.2 Matrix Gaussian Integrals; 2.2.2.1 Application of Wick's Theorem to Matrix Integrals; 2.2.2.2 From Graphs to Maps; 2.3 Generating Functions of Maps and Matrix Integrals 2.3.1 Generating Functions for Closed Maps2.3.1.1 Connected Maps; 2.3.1.2 Topological Expansion: Maps of Given Genus; 2.4 Maps with Boundaries or Marked Faces; 2.4.1 One Boundary; 2.4.2 Several Boundaries; 2.4.3 Topological Expansion for Bounded Maps of Given

Sommario/riassunto

The problem of enumerating maps (a map is a set of polygonal "countries" on a world of a certain topology, not necessarily the plane or the sphere) is an important problem in mathematics and physics, and it has many applications ranging from statistical physics, geometry, particle physics, telecommunications, biology, ... etc. This problem has been studied by many communities of researchers, mostly combinatorists, probabilists, and physicists. Since 1978, physicists have invented a method called "matrix models" to address that problem, and many results have been obtained. Besides, another important problem in mathematics and physics (in particular string theory), is to count Riemann surfaces. Riemann surfaces of a given topology are parametrized by a finite number of real parameters (called moduli), and the moduli space is a finite dimensional compact manifold or orbifold of complicated topology. The number of Riemann surfaces is the volume of that moduli space. More generally, an important problem in algebraic geometry is to characterize the moduli spaces, by computing not only their volumes, but also other characteristic numbers called intersection numbers. Witten's conjecture (which was first proved by Kontsevich), was the assertion that Riemann surfaces can be obtained as limits of polygonal surfaces (maps), made of a very large number of very small polygons. In other words, the number of maps in a certain limit, should give the intersection numbers of moduli spaces. In this book, we show how that limit takes place. The goal of this book is to explain the "matrix model" method, to show the main results obtained with it, and to compare it with methods used in combinatorics (bijective proofs, Tutte's equations), or algebraic geometry (Mirzakhani's recursions). The book intends to be self-contained and accessible to graduate students, and provides comprehensive proofs, several examples, and gives the general formula for the enumeration of maps on surfaces of any topology. In the end, the link with more general topics such as algebraic geometry, string theory, is discussed, and in particular a proof of the Witten-Kontsevich conjecture is provided.