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## Sommario/riassunto

This monograph deals with the mathematics of extending given partial data-sets obtained from experiments; Experimentalists frequently gather spectral data when the observed data is limited, e.g., by the precision of instruments; or by other limiting external factors. Here the limited information is a restriction, and the extensions take the form of full positive definite function on some prescribed group. It is therefore both an art and a science to produce solid conclusions from restricted or limited data. While the theory of is important in many areas of pure and applied mathematics, it is difficult for students and for the novice to the field, to find accessible presentations which cover all relevant points of view, as well as stressing common ideas and interconnections. We have aimed at filling this gap, and we have stressed hands-on-examples.

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