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|-------------------------|---|
| 1. Record Nr.           | UNIBAS000016487   |
| Autore                  | Hurewicz, Witold  |
| Titolo                  | Lectures on ordinary differential equations / Witold Hurewicz |
| Pubbl/distr/stampa      | New York : Dover, 1990  |
| ISBN                    | 0-486-66420-1   |
| Descrizione fisica      | XVII, 122 p. ; 22 cm.   |
| Collana                 | Dover books on advanced mathematics                           |
| Disciplina              | 515.352   |
| Soggetti                | Equazioni differenziali                                       |
| Lingua di pubblicazione | Inglese   |
| Formato                 | Materiale a stampa  |
| Livello bibliografico   | Monografia  |
- 
- |                    |  |
|--------------------|--|
| 2. Record Nr.      | UNINA9910758501003321  |
| Autore             | Kurasov P  |
| Titolo             | Spectral Geometry of Graphs // by Pavel Kurasov  |
| Pubbl/distr/stampa | Berlin, Heidelberg : , : Springer Berlin Heidelberg : , : Imprint :<br>Birkhäuser, , 2024  |
| ISBN               | 9783662678725<br>3662678721  |
| Edizione           | [1st ed. 2024.]  |
| Descrizione fisica | 1 online resource (0 pages)  |
| Collana            | Operator Theory: Advances and Applications, , 2296-4878 ; ; 293  |
| Disciplina         | 006.3843<br>530.12   |
| Soggetti           | Quantum computers<br>Mathematical analysis<br>System theory<br>Control theory<br>Mathematical optimization<br>Calculus of variations<br>Quantum Computing<br>Analysis<br>Systems Theory, Control<br>Calculus of Variations and Optimization<br>Teoria espectral (Matemàtica)<br>Operadors diferencials |

Mètodes gràfics  
Llibres electrònics

Lingua di pubblicazione

Inglese

Formato

Materiale a stampa

Livello bibliografico

Monografia

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## Sommario/riassunto

This open access book gives a systematic introduction into the spectral theory of differential operators on metric graphs. Main focus is on the fundamental relations between the spectrum and the geometry of the underlying graph. The book has two central themes: the trace formula and inverse problems. The trace formula is relating the spectrum to the set of periodic orbits and is comparable to the celebrated Selberg and Chazarain-Duistermaat-Guillemin-Melrose trace formulas.

Unexpectedly this formula allows one to construct non-trivial crystalline measures and Fourier quasicrystals solving one of the long-standing problems in Fourier analysis. The remarkable story of this mathematical odyssey is presented in the first part of the book. To solve the inverse problem for Schrödinger operators on metric graphs the magnetic boundary control method is introduced. Spectral data depending on the magnetic flux allow one to solve the inverse problem in full generality, this means to reconstruct not only the potential on a given graph, but also the underlying graph itself and the vertex conditions. The book provides an excellent example of recent studies where the interplay between different fields like operator theory, algebraic geometry and number theory, leads to unexpected and sound mathematical results. The book is thought as a graduate course book

where every chapter is suitable for a separate lecture and includes problems for home studies. Numerous illuminating examples make it easier to understand new concepts and develop the necessary intuition for further studies.

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